Tor André Myrvoll

On the Use of Copulas in Channel Modeling for Wireless Communications

Communication Systems, SINTEF ICT

Outline

- Channel modeling for wireless communications
- Copulas
- Tail dependencies in real channels
- Conclusions and further work

Wireless Communications 101

A wireless communications system is usually described using the following simple equation

$$y(t) = \int_0^\infty h(t,\tau)x(t-\tau)d\tau + w(t)$$

or in the discrete time case

$$y(n) = \sum_{k=0}^{\infty} h(n,k)x(n-k) + w(n)$$

The function $h(n,k)$ is known as the *time-variant impulse response*, and $w(n)$ is additive noise. In general $h(n,k)$ is a complex entity, representing both *attenuation* and *phase*. We assume that the noise is i.i.d.

Wireless Communications 101

The channel model can usually be separated into three constituents:

- *A determistic model* for the free space path loss, usually on the form $L = C \cdot d^{-n}$, where $n = 2$ for free space.
- *A shadowing model* that describes effect from obstacles blocking and thus affecting the radio signals.
- *A fading model* that describes the effects of constructive and destructive self-interference in a scattering environment.

In addition we make a distinction between flat,

$$y(n) = h(n)x(n) + w(n)$$

and frequency selective fading,

$$y(n) = \sum_{k=0}^{\infty} h(n,k)x(n-k) + w(n)$$

Wireless Communications 101

- In a practical communication system we can estimate $h(n)$ and compensate for the phase rotation, leaving us with the fluctuating attenuation, $|h(n)|$.
- The usual model assumption is that $h(n)$ is a Gaussian process with some bandwidth W_D – the Doppler spread.
- Under the Gaussian process assumption, $|h(n)|$ will follow a Rayleigh distribution if zero-mean:

$$f_X(x;\sigma) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$$

- If non-zero mean, meaning that a line-of-sight component is present, $|h(n)|$ will follow a Rician distribution.

$$f_X(x;\nu,\sigma) = \frac{x}{\sigma^2} e^{-\frac{x^2+\nu^2}{2\sigma^2}} I_0\left(\frac{x\nu}{\sigma^2}\right)$$

Wireless Communications 101

Rayleigh and Rician fading are developed from first principles using assumptions about rich scattering environments. In practice though, some channel measurements made in special environments like indoor office spaces does not fit this model, and alternative distributions have been used successfully

- Weibull:

$$f_X(x; \lambda, k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}$$

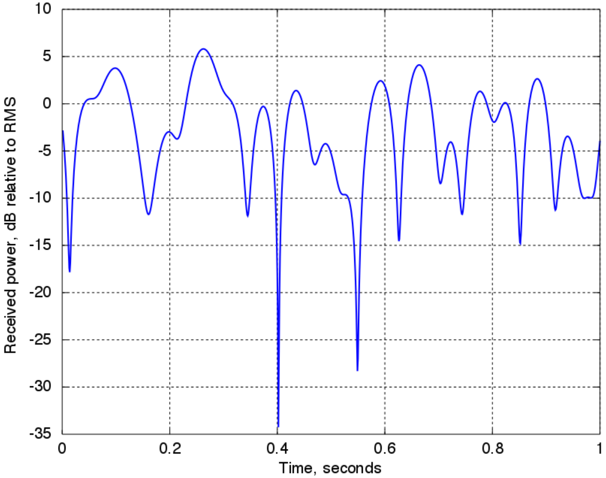
- Nakagami:

$$f_X(x; m, \Omega) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} e^{-\frac{m}{\Omega}x^2}$$

- Log-normal:

$$f_X(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\log x - \mu)^2}{2\sigma^2}}$$

Wireless Communications 101



Fading and Capacity

In communication systems one of the most important quantities is *Capacity*

$$C = \log\left(1 + \frac{P}{N_0}\right)$$

where P is the received signal power and N_0 is the noise power. For a fading channel we rewrite the above definition using

$$\gamma(n) = |h(n)|\sqrt{\frac{P}{N_0}}$$

where $\gamma^2(n)$ is the *instantaneous SNR*,

$$C(n) = \log\left(1 + \gamma^2(n)\right)$$

Fading and Capacity

Using the instantaneous capacity to describe the capacity of the channel is meaningless, as there exists no $0 < C <= C(n)\ \forall n$. Instead we define the *ergodic* and *outage* capacities.

- Outage capacity C_ϵ at target probability ϵ

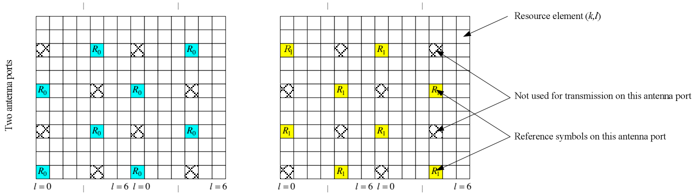
$$\epsilon = P_\Gamma\left(\log(1+\gamma^2) < C_\epsilon\right)$$

- Ergodic capacity under receiver CSI:

$$C = \mathbb{E}_\Gamma\left[\log(1+\gamma^2)\right]$$

Diversity

When in a fading environment one should make use of *diversity* whenever possible. Having diversity means that multiple *channels* or *signal paths* are available either over time, frequency or space. If the fading is independent the probability of outage is lowered and the outage capacity increases.



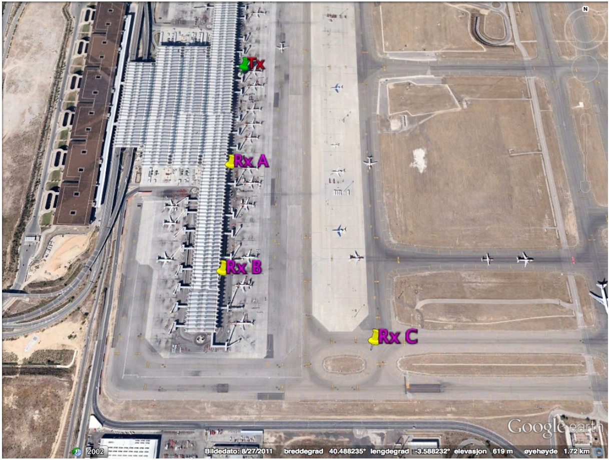
Diversity and tail dependence

Diversity works when the fading across channels is independent, but most importantly, deep fades should occur independently. Other behaviour would entail *tail dependence*, described by the upper and lower tail dependence, λ_l and λ_u:

$$\lambda_l = \lim_{q \to 0} P(X_2 \leqslant F_2^{-1}(q) | X_1 \leqslant F_1^{-1}(q))$$
$$\lambda_u = \lim_{q \to 1} P(X_2 > F_2^{-1}(q) | X_1 > F_1^{-1}(q))$$

Note that $\lambda_l = \lambda_u = 0$ for linearly dependent normal distributions.
It is of great interest whether tail dependecies occur in real channels. To investigate the matter we use a dataset recorded at the Barajas airport outside of Madrid, Spain.

Channel measurements



SINTEF NTNU

Channel measurements

Channel measurements

The channel measurements used in this work have been performed at the Barajas airport in Madrid, Spain, in conjunction with the development of the AeroMACS system for airport surface communications.

- The measurements includes Non-Line-Of-Sight (NLOS) as well as Line-Of-Sight (LOS) conditions.
- The channel sounding is based on a linear frequency modulated chirp that sweeps over a bandwidth of 50 MHz centered at 5125 MHz every 297 s approximately 3367 chirps per second.
- The duration of each chirp is 100 μs.

Channel measurements

A *chirp* is a signal on the form

$$c(t) = e^{i\pi k t^2}$$

Processing the received chirp using a series of multiplication and convolution we obtain

$$H(kt) = \sqrt{k} e^{-i\frac{\pi}{4}} \left[\left(h(t) \circledast c(t) \right) c^{\star}(t) \right] \circledast c(t)$$

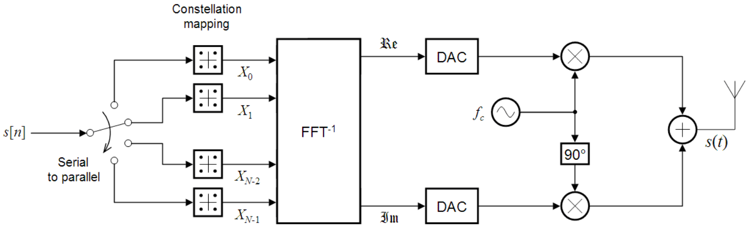
The function $H(kt)$ is snapshop of the *transfer function*

$$H(t_s, f) = \int_{-\infty}^{\infty} h(t, \tau) e^{-i2\pi f \tau} d\tau$$

at time t_s, with frequency and time interchanged.

Channel measurements

The transfer function $H(t, f)$ is directly useful when considering *multicarrier* systems, for example Orthogonal frequency-division multiplexing (OFDM) or Filter Bank Multi Carrier (FBMC).



Copulas

Assume a finite set of random variables $\{X_n\}$, with continuous marginal distributions $\{F_{x_n}(x;\Lambda_n)\}$. Then there exists a copula

$$C : [0,1]^N \rightarrow [0,1],$$

so that the joint distribution can be written,

$$F\left(x_1,\ldots x_N;\Lambda\right) = C\left(F(x_1;\Lambda_1),\ldots,F(x_N;\Lambda_N)\right)$$

There are many families of copulas in the literature, but we will focus on the archimedean copula.

Copulas

A d-dimensional copula C is called Archimedean if for some generator ψ it has distribution:

$$C\left(u_1, \ldots, u_d\right) = \psi^{-1}\{\psi(u_1) + \cdots + \psi(u_d)\}, \quad \forall \mathbf{u} \in [0,1]^d$$

where $\psi : [0,1] \to [0,\infty]$ is the inverse generator with $\psi^{-1}(0) = \inf\{t : \psi(t) = 0\}$.
An Archimedean generator is a continuous, decreasing function that satisfies the following conditions:

1. $\psi(0) = 1$
2. $\psi(\infty) = lim_{t \to \infty} \psi(t) = 0$
3. ψ is strictly decreasing on $[0, \inf\{t : \psi(t) = 0\}]$

Copulas

The coefficient of upper tail dependence is defined as:

$$\lambda_u^{1,\dots,h|h+1,\dots,d} = \lim_{t \to 0^+} \frac{\sum_{i=1}^{d} \left(\binom{d}{d-i} i (-1)^i \left[\psi^{-1'}(it) \right] \right)}{\sum_{i=1}^{d-h} \left(\binom{d-h}{d-h-i} i (-1)^i \left[\psi^{-1'}(it) \right] \right)}. \tag{1}$$

The coefficient of lower tail dependence is defined as:

$$\lambda_l^{1,\dots,h|h+1,\dots,d} = \lim_{t \to \infty} \frac{d}{d-h} \frac{\psi^{-1'}(dt)}{\psi^{-1'}((d-h)t)}. \tag{2}$$

Copulas

Family	ψ	ψ^{-1}
Clayton	$(1+t)^{-\frac{1}{\rho}}$	$(s^{-\rho}-1)$
Frank	$-\frac{1}{\rho}\ln\left[1-e^{-t}(1-e^{-\rho})\right]$	$-\ln\frac{e^{-s\rho}-1}{e^{-\rho}-1}$
Gumbel	$e^{-t^{\frac{1}{\rho}}}$	$(-\ln s)^{\rho}$

- Clayton: The Clayton copula is asymmetric with stronger dependency in the lower tail.
- Gumbel: The Gumbel copula is asymmetric with stronger dependency in the higher tail
- Frank: The Frank copula is a symmetric copula.

Copulas

There are exact non-linear transformations between the copula parameter ρ and Kendall's rank correlation τ for the Clayton, Frank and Gumbel copulaeå

$$\tau = \begin{cases} \frac{\rho}{\rho+2}, & \text{Clayton Model} \\ 1 + \frac{4D_1(\rho)-1}{\rho}, & \text{Frank Model} \\ \frac{(\rho-1)}{\rho}, & \text{Gumbel Model.} \end{cases} \quad (3)$$

where the Debye function of order one given by

$$D_1 = \frac{1}{\rho}\int_0^{\rho} \frac{t}{exp(t)-1} dt. \quad (4)$$

Kendall τ rank correlation for pairs is given by
$\tau := \mathbb{P}\mathrm{r}\left[(X_1 - X_2)(Y_1 - Y_2) > 0\right] - \mathbb{P}\mathrm{r}\left[(X_1 - X_2)(Y_1 - Y_2) < 0\right]$.

Copulas

To detect any tail dependency in our data we will fit a mixture of Clayton, Frank and Gumbel copulas to our data. The mixture weight will *select* the copula best describing the data, and the copula parameter will in turn give us the lower and upper tail dependencies, λ_l and λ_u. The procedure is as follows:

1. Find the distribution F of the channel fading measurements and obtain its parameters using MLE.
2. Use the cumulative distribution F to map measurements $F^{-1} : X \to U$.
3. Obtain the mixture weights $\{w_k\}$ and copula parameters, $\{\rho_C, \rho_F, \rho_G\}$ using MLE.

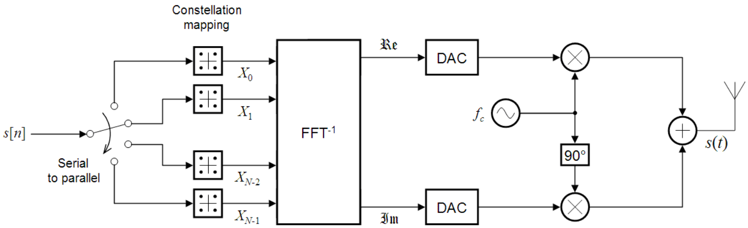
The above procedure is referred to as the Inference Function for Margins (IFM) approach.

Experiments

We chose to use one second of data obtained from the measurements between Tx and RxB, all in all 3367 vectors of 1024 channel measurements – a total bandwidth of 50 MHz.

Experiments

Each channel measurement corresponds to a 50 kHz channel. We will use the centre 256 samples, all in all a bandwidth of 12.5 MHz, to do our experiments, envisioning a multicarrier system.



Experiments

Three experiments are performed:

- Pairwise tail dependency
- Tail dependency in channels with spacing four
- Tail dependency in groups of four contiguous channels

Experiments

The distribution of the channels measurements were selected as follows:

- For each channel index, the following distributions were fitted: Rayleigh, Rice, Weibull, Nakagami and log-normal.
- The Akaike Information Criterion was then used to order the distributions

Using the above procedure it was determined to use the Rice distribution.

Experiments

We then followed the Inference Function for Margins procedure:

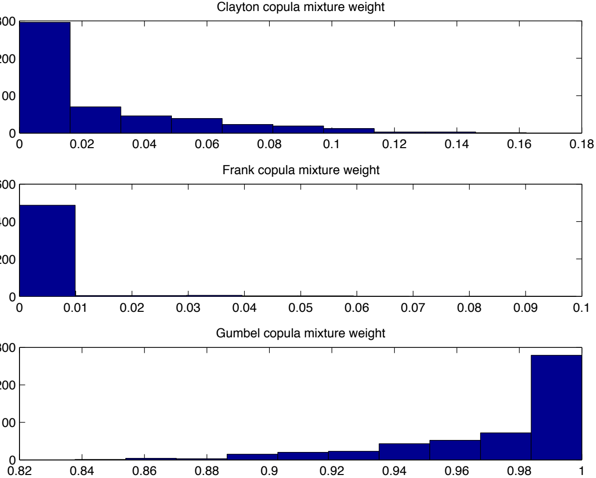
1. For each channel index c, estimate the Rice parameters $\hat{\nu}, \hat{\sigma}$.
2. For each channel index c, the channel measurements were mapped to $[0, 1]$ using the cumulative Rice distribution

$$u_n^c = F(s_n^c; \hat{\nu}, \hat{\sigma}) \tag{5}$$

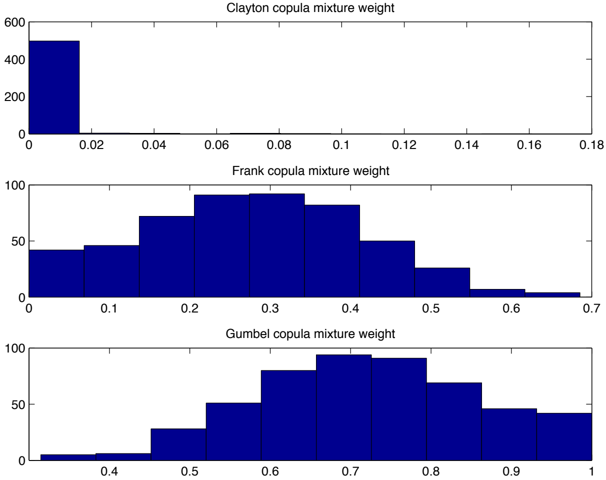
3. The copula mixture weights and parameters were then estimated

$$\begin{aligned} &\left[\widehat{w}_1, \ldots, \widehat{w}_{k-1}, \widehat{\rho}^F, \widehat{\rho}^G, \widehat{\rho}^C\right] \\ &= \arg\max \prod_{n=1}^{N} \sum_{k=1}^{3} w_k c \left(u_n^{(1)}, \ldots, u_n^{(d)}; \rho^F, \rho^G, \rho^C\right) \end{aligned} \tag{6}$$

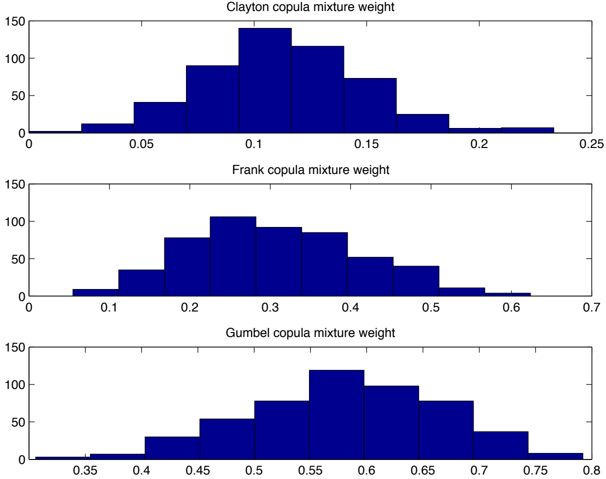
Experiments: Neighboring channels



Experiments: Every four channels

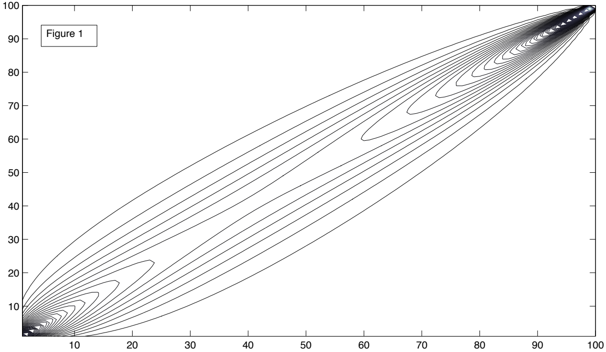


Experiments: Quadruplets

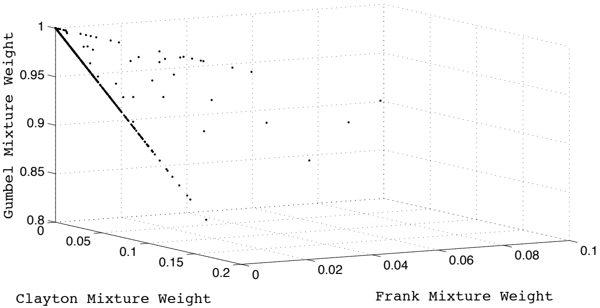


SINTEF NTNU

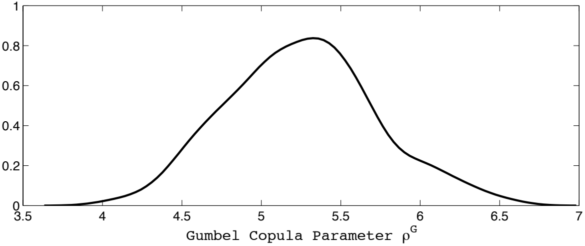
Experiments: Copula contour plot



Experiments: Mixture weights

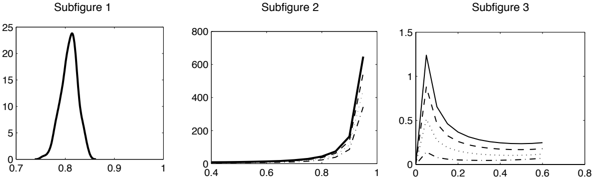


Experiments: Gumbel copula parameter



Experiments: Tail dependence

Subfigure 1 shows the strength of the rank correlation between amplitudes in adjacent frequency bands, subfigure 2 shows the upper tail dependence as a function of exceedance threshold as $q \uparrow 1$ and subfigure 3 shows an identical quantity for lower tail dependence as $q \downarrow 0$.



Conclusions

- The data shows clear upper tail dependency for all three experiments.
- No lower tail dependency detected. Possible explanations:
  - The dependency is "buried" in the noise floor.
  - There is no lower tail dependency
- The upper tail dependence may be explained by a NLOS-S scenario, that is, a scenario where on in general we have non-line-of-sight, but on occation we get strong specular transmit paths.

Internationally outstanding