

## Violations of Uncovered Interest Rate Parity and International Exchange Rate Dependences

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Matthew Ames <sup>1</sup>, Guillaume Bagnarosa <sup>2,1</sup>, Gareth W. Peters <sup>1,3,4</sup>

- 1. Department of Statistical Science, University College London
  - 2. ESC Rennes School of Business
    - 3. CSIRO, Australia
  - 4. Oxford-Man Institute, Oxford University, UK

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## Impact of the Carry Trade on Japanese Yen

"Borrowing yen to buy higher-yielding assets hasn't been this popular since the global financial crisis. That's a rare piece of good news for the Bank of Japan's bid to achieve 2 percent inflation."

Bloomberg (March 16, 2015)

"The BOJ's lending facility promotes yen carry trades, and that's contributing to yen declines. The trend for a weak yen versus the dollar is likely to continue." Takashi Shiono, Economist at Credit Suisse, Tokyo (March 16, 2015)

## Impact of the Carry Trade on Worldwide Central Banks

"...Add a resurgent carry trade to the list of things keeping Reserve Bank Governor Glenn Stevens from getting a weaker Aussie dollar..." Bloomberg (April 19, 2015)

"...Because the kiwi has been put on a pedestal as the carry trade of choice over the last six months, those comments (by the Reserve Bank of New Zealand Assistant Governor John McDermott) have had a big impact this morning..." A. Myers, Strategist at Credit Agricole (Apr 23, 2015)

"What essentially has happened is the Bank of Thailand has joined the currency war... The central bank has reduced the appeal of the baht from the carry perspective."

J. Cavenagh, Strategist at Westpac Banking Corp (May 18, 2015)

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## Covered Interest Rate Parity

This relation states that the price of a forward rate can be expressed as follows:

$$F_t^T = e^{(r_t - r_t^f)(T - t)} S_t \tag{1}$$

where  $F_t^T$  and  $S_t$  denote respectively the forward and the spot prices at time t.

## Uncovered Interest Rate Parity

If we assume the forward price is a martingale under the risk neutral probability  $\mathbb{Q}$ , then the fair value of the forward contract at time *t* equals:

$$\mathsf{E}_{\mathbb{Q}}[S_{\mathcal{T}}|\mathcal{F}_t] = \mathsf{F}_t^{\mathcal{T}} \tag{2}$$

where  $\mathcal{F}_t$  is the filtration associated to the stochastic process  $S_t$ .

## Uncovered Interest Rate Parity

Replacing the expression (2) in the relation (1) leads to the UIP equation:

$$E_{\mathbb{Q}}\left[\frac{S_T}{S_t}\middle|\mathcal{F}_t\right] = \frac{F_t^T}{S_t} = e^{(r_t - r_t^f)(T - t)}$$
(3)

## Uncovered Interest Rate Parity Puzzle

- Well-documented empirical finding that high interest rate currencies tend to appreciate relative to low interest rate currencies
  - violating Uncovered Interest rate Parity (UIP)

## What is the Carry Trade?

- A simple investment strategy:
  - sell a basket of low interest rate currencies (e.g. Japanese yen, Swiss franc,...)
  - buy a basket of high interest rate currencies (e.g. Australian dollar, New Zealand dollar,...).
- Numerous empirical studies -Hansen and Hodrick 1980; Fama 1984; Engel 1984; Lustig and Verdelhan 2007; Brunnermeier, Nagel, and Pedersen 2008; Menkhoff et al. 2012
  - have previously demonstrated that investors can actually earn profits on average using this strategy.

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## Carry Trade Risk/Return



Cumulative monthly log returns of long and short baskets, which combine to make the carry portfolio. Buy: AUD, NZD, NOK, GBP, CAD. Sell: JPY, CHF, EUR.

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## Carry Trade Risk/Return: 8 Currencies



Monthly return distribution for carry trade portfolio using 8 Currencies.

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## Carry Trade Strategy

- Implemented by numerous hedge funds in large size and with high leverage, see Galati, Heath, and McGuire 2007; Fong 2013.
- Also systematic indices sold by investment banks, e.g. SGI FX G10 Carry Trade, DB G10 Currency Harvest or the Barclays Optimized Currency Carry ETN...

However, detection of strategy inflows is not obvious

- as pointed out by Galati, Heath, and McGuire 2007
- generally only verified for downside markets and unwinding periods of the speculative positions.

## Carry Trade Strategy

We postulate that the levered construction, as well as the unwinding of the carry trade portfolios, should influence:

- the individual dynamic of the exchange rates
- more importantly the currency commonalities and thus their dependence structure
- the volumes traded on these currencies and more specifically the speculative volume executed in the market
- finally, the dependence between individual speculative volumes

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## Speculative Volume and Asset Dependences

- Another strand of the literature has shown a link between the asset price variance and the volume traded on the same asset, see He and Velu 2014; Hasbrouck and Seppi 2001; Ané and Geman 2000; Tauchen and Pitts 1983; Clark 1973.
- The Common factor MDH Model (Hasbrouck and Seppi 2001) as well as the multivariate version (He and Velu 2014) are built around the following assumptions:

(4)

## Speculative Volume and Asset Dependences

The price of asset *i*:

$${m P}_i = rac{1}{J}\sum_{j=1}^J {m P}_{ij}^\star$$

where  $P_{ij}^{\star}$  denotes investor j's forecast for the price of asset i.

Furthermore, the volume traded for each asset i can be written as

$$V_{i} = \frac{c}{2} \sum_{j=1}^{J} \|\Delta P_{ij}^{\star} - \Delta P_{i}\|$$
(5)

Where  $\Delta P_i$  is the market price change, which is assumed to be

$$\Delta P_i = \frac{1}{J} \sum_{j=1}^{J} \Delta P_{ij}^{\star} \tag{6}$$

Finally, they assume that trader j's incremental price forecast for asset i can be expressed as a combination of asset-specific and cross-asset components

$$\Delta P_{ij}^{\star} = \nu_i' + \psi_{ij}' + \nu_i^{C} + \psi_{ij}^{C} \,. \tag{7}$$

## Speculative Volume and Asset Dependences

## **Our contributions:**

- Firstly, we show that a large part of the currencies' speculative volumes are driven by the carry trade (and potentially other well known speculative strategies, e.g. commodity driven).
- We show that speculative volumes have more explanatory power than price index changes for covariance and tail dependences among currencies (Multivariate MDH model).

## Speculative Volume and Asset Dependences

## Our contributions:

- We propose a dynamic covariance model taking into consideration the stylized facts.
- We demonstrate that not only the downward extremal dependence among currencies, but also the upward equivalent are impacted by speculator behaviour.
- Also, we extended our model to 34 currencies (mixing developed and developing countries).

## Data and Carry Trade Portfolio Construction

- 8 currencies as studied in Brunnermeier, Nagel, and Pedersen 2008
- High Interest Rate Basket: Australia (AUD), Canada (CAD), New Zealand (NZD), Norway (NOK), United Kingdom (GBP)
- Low Interest Rate Basket: Japan (JPY), Switzerland (CHF), Euro (EUR).

## Data and Carry Trade Portfolio Construction

Daily settlement prices:

- spot exchange rate
- associated 1 month forward contract
- 02/01/1989 29/01/2014

Weekly speculative open interest ratio:

- SPEC = (NNCFP / Open Interest)
- 20/06/2006 29/01/2014
- Norway (NOK) SPEC unavailable

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## High Interest Rate Basket: Spot Prices



Spot prices normalised by spot price on 02/01/1999.

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## Low Interest Rate Basket: Spot Prices



Spot prices normalised by spot price on 02/01/1999.

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## Price Factors: $DOL + HML_{FX}$



**Left Subplot:** Loadings of the 1st PC of Currency Returns. **Right Subplot:** Loadings of the 2nd PC of Currency Returns.

## Price Factors: $DOL + HML_{FX}$

Suggested by Lustig and Verdelhan 2007:

- *DOL*: average excess return of all currencies against the dollar
- *HML<sub>FX</sub>*: High interest rate basket return Low interest rate basket return

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## High Interest Rate Basket: SPEC



 $\mathsf{SPEC} = \mathsf{NNCFP} \ / \ \mathsf{Open} \ \mathsf{Interest}$ 

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## Low Interest Rate Basket: SPEC



 $\mathsf{SPEC} = \mathsf{NNCFP} \ / \ \mathsf{Open} \ \mathsf{Interest}$ 

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## Speculative Open Interest (SPEC)



Loadings of the 1st PC of Speculative volume ratio (SPEC).

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## Currency Returns and SPEC

# How does SPECulative volume affect currency returns:

- 🗸 mean
  - covariance
  - tail dependence

Conclusions

## Mean regression: Individual currency returns $\sim$ SPEC

	AUD	CAD	CHF	EUR	GBP	JPY	NZD
Constant	0.000	0.002	-0.001	-0.001	0.001	0.001	-0.001
	(0.256)	(0.376)	(0.443)	(0.975)	(0.060)	(0.347)	(0.545)
DOL	-0.509	-0.313	-0.346	-0.383	-0.318	-0.058	-0.522
	(0.000)	(0.004)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
HML <sub>FX</sub>	0.308	0.202	-0.591	-0.287	-0.109	-0.572	0.314
	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
AUD	-0.001	0.001	-0.002	0.002	-0.001	0.001	0.001
	(0.264)	(0.586)	(0.921)	(0.423)	(0.365)	(0.115)	(0.825)
CAD	0.001	-0.005	-0.001	0.001	0.001	0.001	0.001
	(0.863)	(0.760)	(0.336)	(0.514)	(0.005)	(0.403)	(0.294)
CHF	0.001	0.004	0.001	-0.001	-0.002	0.001	-0.001
	(0.561)	(0.470)	(0.387)	(0.794)	(0.065)	(0.882)	(0.470)
EUR	-0.001	-0.002	-0.001	-0.001	0.005	-0.002	0.001
	(0.756)	(0.478)	(0.032)	(0.495)	(0.390)	(0.963)	(0.939)
GBP	0.001	0.005	0.001	-0.003	-0.004	0.003	-0.001
	(0.065)	(0.168)	(0.127)	(0.492)	(0.023)	(0.513)	(0.769)
JPY	-0.004	-0.001	0.001	0.001	0.005	-0.005	0.001
	(0.764)	(0.030)	(0.021)	(0.005)	(0.504)	(0.330)	(0.577)
NZD	0.001	-0.002	0.004	0.001	-0.001	-0.004	-0.001
	(0.560)	(0.088)	(0.590)	(0.804)	(0.271)	(0.044)	(0.712)
$R^2(DOL,HML_{FX})$	0.916	0.675	0.804	0.805	0.596	0.573	0.903
$R^2(DOL,HML_{FX},SPEC)$	0.920	0.685	0.807	0.811	0.611	0.587	0.904

Individual currency returns  $\sim DOL + HML_{FX} + SPEC$ .

Numbers in parentheses show Newey and West (1987) HAC p-values.

Conclusions

## Currency Returns and SPEC

# How does SPECulative volume affect currency returns:

- mean
- ✓ covariance
  - tail dependence

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• Firstly, we de-trend the currency returns using the mean regression (as before) to get residuals  $e_i = y_i - \beta x_i$ ,

where  $\mathbf{x}_i = [DOL(i), HML_{FX}(i)]^T$  or  $\mathbf{x}_i = [DOL(i), HML_{FX}(i), SPEC(i)]^T$ or  $\mathbf{x}_i = [DOL(i), HML_{FX}(i), SPEC(i), SPEC(i) * SPEC(i)]^T$ .

• We model the covariance matrix for  $e_i$ , conditional on  $x_i$  given by,

$$\mathbb{E}[\boldsymbol{e}_{i}\boldsymbol{e}_{i}^{T}|\boldsymbol{x}_{i}] = \boldsymbol{B}\boldsymbol{x}_{i}\boldsymbol{x}_{i}^{T}\boldsymbol{B}^{T} + \boldsymbol{\Psi}$$
(8)

• Convenient to use the following random-effects representation:

$$\begin{split} & \boldsymbol{e}_{i} = \gamma_{i} \times \boldsymbol{B}\boldsymbol{x}_{i} + \boldsymbol{\epsilon}_{i} \\ & \mathbb{E}[\boldsymbol{\epsilon}_{i}] = 0 \quad , \quad Cov(\boldsymbol{\epsilon}_{i}) = \boldsymbol{\Psi} \\ & \mathbb{E}[\gamma_{i}] = 0 \quad , \quad Var[\gamma_{i}] = 1 \quad , \quad \mathbb{E}[\gamma_{i} \times \boldsymbol{\epsilon}_{i}] = 0 \end{split}$$

$$\mathbb{E}[\boldsymbol{e}_{i}\boldsymbol{e}_{i}^{T}|\boldsymbol{x}_{i}] = \mathbb{E}[\gamma_{i}^{2}\boldsymbol{B}\boldsymbol{x}_{i}\boldsymbol{x}_{i}^{T}\boldsymbol{B}^{T} + \gamma_{i}(\boldsymbol{B}\boldsymbol{x}_{i}\boldsymbol{\epsilon}_{i}^{T} + \boldsymbol{\epsilon}\boldsymbol{x}_{i}^{T}\boldsymbol{B}^{T}) + \boldsymbol{\epsilon}_{i}\boldsymbol{\epsilon}^{T}] \\ = \boldsymbol{B}\boldsymbol{x}_{i}\boldsymbol{x}_{i}^{T}\boldsymbol{B}^{T} + \boldsymbol{\Psi}$$

$$(9)$$

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Estimate coefficients via EM:

Iteratively maximising the complete data log-likelihood
 *l*(*B*, Ψ) = log *p*(*E*|*B*, Ψ, *X*, γ), which is obtained from the multivariate normal density given by

$$-2I(\boldsymbol{B},\boldsymbol{\Psi}) = np\log(2\pi) + n\log|\boldsymbol{\Psi}| + \sum_{i=1}^{n} (\boldsymbol{e}_{i} - \gamma_{i}\boldsymbol{B}\boldsymbol{x}_{i})^{T}\boldsymbol{\Psi}^{-1}(\boldsymbol{e}_{i} - \gamma_{i}\boldsymbol{B}\boldsymbol{x}_{i}).$$
(10)

• We note that the conditional distribution of the random effects given the data and covariates is then conveniently given by a normal distribution in each element according to  $\{\gamma_i | \boldsymbol{E}, \boldsymbol{X}, \boldsymbol{\Psi}, \boldsymbol{B}\} = \mathcal{N}(m_i, v_i)$  with mean  $m_i = v_i(e_i^T \boldsymbol{\Psi}^{-1} \boldsymbol{B} \boldsymbol{x}_i)$  and variance  $v_i = (1 + \boldsymbol{x}_i^T \boldsymbol{B}^T \boldsymbol{\Psi}^{-1} \boldsymbol{B} \boldsymbol{x}_i)^{-1}$  see Hoff and Niu 2011.

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EM algorithm: Expectation Step

- Initialize the parameter matrices,  $\widehat{\Psi}$  and  $\widehat{B}$ .
- Calculate the conditional estimators:

$$m_{i} = \mathbb{E}\left[\Gamma_{i}|\widehat{\Psi},\widehat{B}, e_{i}\right]$$
  

$$v_{i} = \mathbb{V}ar\left[\Gamma_{i}|\widehat{\Psi},\widehat{B}, e_{i}\right]$$
(11)

- Construct new matrices  $\widetilde{X}$  and  $\widetilde{E}$  based on the data  $y_{1:n}$  and covariates  $x_{1:n}$ .
  - $\widetilde{\boldsymbol{E}}$  is the 2*n* × 1 matrix given by  $(\boldsymbol{E}^{T}, \mathbf{0} \times \boldsymbol{E}^{T})^{T}$
  - ►  $\widetilde{X}$  is a  $2n \times d$  matrix with *i*-th row given by  $m_i x_i$  and whose (n + i)-th is  $\sqrt{v_i x_i}$ .

EM algorithm: Maximisation Step

• Evaluate the updated model parameters via the following least squares solutions for updated  $\widehat{\Psi}$  and  $\widehat{B}$  according to

$$\widehat{\boldsymbol{B}} = \widetilde{\boldsymbol{E}}^{T} \widetilde{\boldsymbol{X}} \left( \widetilde{\boldsymbol{X}}^{T} \widetilde{\boldsymbol{X}} \right)^{-1}$$

$$\widehat{\boldsymbol{\Psi}} = \frac{1}{n} \left( \widetilde{\boldsymbol{E}} - \widetilde{\boldsymbol{X}} \widehat{\boldsymbol{B}} \right)^{T} \left( \widetilde{\boldsymbol{E}} - \widetilde{\boldsymbol{X}} \widehat{\boldsymbol{B}} \right)$$
(12)

• repeat the above procedure until convergence.

Non-Baseline Variance 
$$\% = 100 \times \frac{trace(\boldsymbol{B}\boldsymbol{X}_{(0.5)}\boldsymbol{X}_{(0.5)}^{T}\boldsymbol{B}^{T})}{trace(\boldsymbol{B}\boldsymbol{X}_{(0.5)}\boldsymbol{X}_{(0.5)}^{T}\boldsymbol{B}^{T}) + trace(\Psi)}$$

- The proportion of covariation in the covariance regression attributed to the factors relative to the total second order explanatory power of the covariance regression on each 125 week sliding window.
- This measure focuses on the covariance explained when X takes its median value, denoted  $X_{(0.5)}$ .

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#### Conclusions

### Covariance regression: Multivariate Currency Returns $\sim$ SPEC



 $DOL + HML_{FX}$  vs  $DOL + HML_{FX} + SPEC + SPEC \times SPEC$ . 125 week lookback periods.

Conclusions

## Currency Returns and SPEC

# How does SPECulative volume affect currency returns:

- mean
- covariance
- $\checkmark$  tail dependence

# Modelling the Multivariate Exchange Rate Returns of the Baskets

We consider the two stage estimation procedure known as the inference function for margins (IFM) technique as studied in Joe and Xu 1996.

Stage 1: estimation of suitable heavy tailed marginal models

Stage 2: followed by estimation of the dependence component of the multivariate model

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## Modelling the Marginals - LogNormal?

## We initially considered LogNormal margins:



Percentage of rejections using the Kolmogorov-Smirnov test at 5% significance level for log normal margins

## Modelling the Marginals - Log Generalized Gamma

Since a number of the currencies appearing in the high interest rate basket failed K-S tests for a significant portion of days:

• We considered a more flexible three parameter model:

Generalized-Gamma distribution

$$f_X(x; k, \alpha, \beta) = \frac{\beta}{\Gamma(k)} \frac{x^{\beta k - 1}}{\alpha^{\beta k}} \exp\left(-\left(\frac{x}{\alpha}\right)^{\beta}\right)$$
(13)

 $\implies$  Log-Generalized-Gamma distribution (l.g.g.d.)

$$f_{Y}(y;k,u,b) = \frac{1}{b\Gamma(k)} \exp\left[k\left(\frac{y-u}{b}\right) - \exp\left(\frac{y-u}{b}\right)\right]$$
(14)

with  $u = \log(\alpha)$ ,  $b = \beta^{-1}$ ,  $supp(y) \in \mathbb{R}$ .

## Modelling the Dependence Structure via Mixture Copulae

We model the dependence between log exchange rate returns using a Clayton-Frank-Gumbel mixture copula:

- asymmetric flexibility in the tails
- automatic model selection

## Definition (Mixture Copula)

A mixture copula is a linear weighted combination of copulae of the form:

$$C_M(\mathbf{u}; \mathbf{\Theta}) = \sum_{i=1}^N w_i C_i(\mathbf{u}; \theta_i)$$
(15)

where 
$$0 \le w_i \le 1 \ \forall i = 1, ..., N$$

and 
$$\sum_{i=1}^{N} w_i = 1$$

- Thus we can combine:
  - ▶ a copula with lower tail dependence
  - ▶ a copula with positive or negative dependence
  - ▶ a copula with upper tail dependence
- to produce a more flexible copula capable of modelling the multivariate log returns of forward exchange rates of a basket of currencies.

## Multivariate Tail Dependence

Using the fitted mixture of copula functions:

- we can extract a measure of the multivariate tail dependence within each basket
  - see De Luca and Rivieccio 2012.

Conclusions

## Multivariate Tail Dependence - Upper

Definition (Generalized Upper Tail Dependence Coefficient)

Let  $X = (X_1, ..., X_d)^T$  be a d dimensional random vector with marginal distribution functions  $F_1, ..., F_d$ . The coefficient of upper tail dependence is defined as:

$$\lambda_{u}^{1,...,h|h+1,...,d} = \lim_{\nu \to 1^{-}} P\left(X_{1} > F^{-1}(\nu), ..., X_{h} > F^{-1}(\nu)|X_{h+1} > F^{-1}(\nu), ..., X_{d} > F^{-1}(\nu)\right)$$
$$= \lim_{t \to 0^{+}} \frac{\sum_{i=1}^{d} \left( \binom{d}{d-i} i(-1)^{i} \left[ \psi^{-1'}(it) \right] \right)}{\sum_{i=1}^{d-h} \left( \binom{d-h}{d-h-i} i(-1)^{i} \left[ \psi^{-1'}(it) \right] \right)}$$

(16)

Conclusions

## Multivariate Tail Dependence - Lower

Definition (Generalized Lower Tail Dependence Coefficient)

Let  $X = (X_1, ..., X_d)^T$  be a d dimensional random vector with marginal distribution functions  $F_1, ..., F_d$ . The coefficient of lower tail dependence is defined as:

$$\lambda_{l}^{1,...,h|h+1,...,d} = \lim_{\nu \to 0+} P\left(X_{1} < F^{-1}(\nu), ..., X_{h} < F^{-1}(\nu)|X_{h+1} < F^{-1}(\nu), ..., X_{d} < F^{-1}(\nu)\right)$$
$$= \lim_{t \to \infty} \frac{d}{d-h} \frac{\psi^{-1'}(dt)}{\psi^{-1'}((d-h)t)}$$
(17)

## Tail Dependence - High Interest Rate Basket

#### VIX vs Tail dependence present in High IR Basket



Comparison of Volatility Index (VIX) with upper and lower tail dependence of the high interest rate basket.

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ΤI	) (	$\sim$	SPI	EC											
	D	OL	HML	σDOL	$\sigma_{HML_{FX}}$	$\sigma_{DOL,HML_{FX}}$	AUD	CAD	CHF	EUR	GBP	JPY	NZD	CROSS	R <sup>2</sup>
Â	H 0.3	143 321)	-0.020 (0.945)	3.166 (0.324)	-0.246 (0.976)	577.011 (0.072)									5.2%
	0.: (0.:	134 390)	0.135 (0.609)	-0.335 (0.920)	-1.715 (0.869)	239.629 (0.378)	0.083 (0.066)	-0.053 (0.160)	0.173 (0.000)	-0.084 (0.101)	-0.070 (0.214)	-0.128 (0.002)	-0.104 (0.007)		23.6%
	0.0 (0.9	014 920)	-0.058 (0.785)	1.640 (0.503)	-2.267 (0.794)	333.003 (0.173)	0.272 (0.000)	0.114 (0.204)	0.036 (0.687)	-0.109 (0.125)	-0.290 (0.006)	-0.137 (0.009)	0.025 (0.675)	*	46%
$\hat{\lambda}$	H 0.0 (0.1	083 707)	0.303 (0.380)	-8.696 (0.051)	26.700 (0.003)	-258.500 (0.488)									14.5%
	-0. (0.9	023 913)	0.030 (0.922)	-2.343 (0.553)	23.177 (0.009)	-13.838 (0.966)	-0.129 (0.017)	0.126 (0.008)	-0.023 (0.679)	0.154 (0.014)	0.046 (0.404)	-0.023 (0.631)	0.008 (0.878)		27.1%
	-0. (0.3	162 317)	-0.032 (0.921)	0.625 (0.863)	16.159 (0.058)	-201.897 (0.576)	-0.173 (0.059)	0.281 (0.011)	-0.014 (0.900)	0.232 (0.042)	0.133 (0.203)	-0.053 (0.534)	-0.020 (0.745)	*	39.4%
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High interest rate respective tail dependences  $\sim DOL + HML_{FX} + DOL^{VOL} + HML_{FX}^{VOL} + DOL \times HML_{FX} + SPEC + SPEC * SPEC$ . Numbers in parentheses show Newey and West (1987) HAC p-values.

## Tail Dependence - Low Interest Rate Basket

#### VIX vs Tail dependence present in Low IR Basket



Comparison of Volatility Index (VIX) with upper and lower tail dependence of the low interest rate basket.

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ΤI	C	$\sim$	SPE	EC											
_	I	DOL	HML <sub>FX</sub>	σDOL	$\sigma_{HML_{FX}}$	$\sigma_{DOL,HML_{FX}}$	AUD	CAD	CHF	EUR	GBP	JPY	NZD	CROSS	R <sup>2</sup>
Â	/ 0 / (0	.249 .223)	0.410 (0.227)	-2.268 (0.609)	31.495 (0.001)	1103.721 (0.008)									11.7%
	-( (0	).006 ).972)	0.155 (0.590)	-1.606 (0.664)	31.478 (0.000)	868.169 (0.007)	-0.083 (0.154)	0.054 (0.236)	0.186 (0.001)	0.002 (0.975)	-0.011 (0.855)	-0.054 (0.278)	0.122 (0.017)		31.7%
_	-( (0	0.061 0.707)	-0.041 (0.859)	2.691 (0.510)	16.269 (0.065)	776.107 (0.023)	-0.105 (0.160)	-0.013 (0.899)	0.219 (0.027)	-0.031 (0.812)	-0.124 (0.214)	-0.109 (0.084)	0.129 (0.024)	*	48.5%
$\hat{\lambda}$	L -0 (0	0.319 0.089)	-0.205 (0.445)	-8.733 (0.000)	-8.918 (0.098)	-1088.660 (0.000)									15.6%
	-( (0	0.264 0.079)	0.056 (0.807)	-7.866 (0.001)	-10.300 (0.186)	-958.776 (0.000)	0.089 (0.052)	0.025 (0.663)	-0.136 (0.002)	0.129 (0.012)	-0.162 (0.004)	-0.040 (0.370)	-0.027 (0.379)		30.9%
	-( (0	0.107 0.418)	-0.172 (0.398)	-8.883 (0.000)	4.468 (0.516)	-710.730 (0.004)	-0.049 (0.403)	-0.017 (0.804)	-0.130 (0.028)	0.149 (0.049)	0.035 (0.637)	0.016 (0.771)	-0.064 (0.141)	*	53.7%
		Lo	ow int	erest	rate r	espective	tail c × <i>HI</i>	lepen ML cv	dence	$\sim 1$	DOL ⊢ SPI	+ HN	1L <sub>FX</sub> SPF(	+	
		N	umbe	rs in	parent	heses sho	w Ne	wey a	and V	Vest (	1987	) HA(	C 2		

p-values.

UIP and the Carry Trade

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Conclusions

## Currency Returns and SPEC



6-month rolling upper tail dependence of low interest rate developed countries (namely JPY, CHF, EUR) compared to net open position on the Swiss franc future contract.

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Outline			
1 Introdu	iction		
2 UIP an	d the Carry Trade		
• Cove	ered Interest Rate Pa	rity	
• Unco	overed Interest Rate	Parity	
• Unco	overed Interest Rate	Parity Puzzle	
What what is a second secon	t Is the Carry Trade?		
3 Specul	ative Volume and Cu	rrency Returns	
• Liter	ature and Our Contr	ibutions	
• Data	)		
• Curr	ency Returns		
Spece	culative Volume (SPE	EC)	
• Mea	n Regression		
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• Tail	Dependence		
4 Conclu	sions		

- A large part of the currencies' speculative volumes is driven by the carry trade.
- Speculative volumes have more explanatory power than price index changes for covariance and tail dependences among currencies.
- Downward and upward extremal dependences among currencies are impacted by speculator behaviour.

Introduction	UIP and the Carry Trade	Speculative Volume and Currency Returns	
Conclusion	S		

- During crisis periods the high interest rate currencies tend to display very significant upper tail dependence.
- High interest rate and low interest rate currency baskets can display periods during which the tail dependence gets inverted, showing construction periods of carry positions.
- Tail dependence can be used to measure the construction and unwinding of speculative positions in assets for which we don't have speculative volume data!

UIP and the Carry Trade

Speculative Volume and Currency Returns

## Questions?



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## Profile Likelihood Plots - Example Day 1



Profile likelihood plots for C-F-G mixture model.

## Profile Likelihood Plots - Example Day 2



Profile likelihood plots for C-F-G mixture model.

## Clayton-Frank-Gumbel Mixture Copula

$$C_{\rho_{C},\rho_{F},\rho_{G}}^{CFG}(\mathbf{u}) = \lambda_{C}(C_{\rho_{C}}^{C}(\mathbf{u})) + \lambda_{F}(C_{\rho_{F}}^{F}(\mathbf{u})) + \lambda_{G}(C_{\rho_{G}}^{G}(\mathbf{u}))$$

$$= \lambda_{C} \times \left(\sum_{i=1}^{d} u_{i}^{-\rho} - d + 1\right)^{-\frac{1}{\rho}}$$

$$+ \lambda_{F} \times -\frac{1}{\rho} \ln \left(1 + \frac{\prod_{i=1}^{d} (e^{-\rho u_{i}} - 1)}{(e^{-\rho} - 1)^{d-1}}\right)$$

$$+ \lambda_{G} \times e^{-\left(\sum_{i=1}^{d} (-\log u_{i})^{\rho}\right)^{\frac{1}{\rho}}}$$

$$(18)$$

## Clayton-Frank-Gumbel Mixture Copula Density

$$c_{\rho_{C},\rho_{F},\rho_{G}}^{CFG}(\mathbf{u}) = \lambda_{C}(c_{\rho_{C}}^{C}(\mathbf{u})) + \lambda_{F}(c_{\rho_{F}}^{F}(\mathbf{u})) + \lambda_{G}(c_{\rho_{G}}^{G}(\mathbf{u}))$$

$$= \lambda_{C} \times \prod_{k=0}^{d-1} (\rho_{k}+1) \left(\prod_{i=1}^{d} u_{i}\right)^{-(1+\rho)} (1+t_{\rho}^{C}(\mathbf{u}))^{\left(-d+\frac{1}{\rho}\right)}$$

$$+ \lambda_{F} \times \left(\frac{\rho}{1-e^{-\rho}}\right)^{d-1} Li_{-(d-1)} \left\{h_{\rho}^{F}(\mathbf{u})\right\} \frac{e^{\left(-\rho\sum_{j=1}^{d} u_{j}\right)}}{h_{\rho}^{F}(\mathbf{u})} \quad (19)$$

$$+ \lambda_{G} \times \rho^{d} e^{\left(-t_{\rho}(\mathbf{u})^{\frac{1}{\rho}}\right)} \frac{\prod_{i=i}^{d} (-\log u_{i})^{\rho-1}}{t_{\rho}(\mathbf{u})^{d} \prod_{i=1}^{d} u_{i}} P_{d,\frac{1}{\rho}}^{G}(t_{\rho}^{G}(\mathbf{u})^{\frac{1}{\rho}})$$

# Clayton-Frank-Gumbel Mixture Copula Density - Continued

#### where

$$t_{\rho}^{C}(\mathbf{u}) = \sum_{i=1}^{d} \psi_{C}^{-1}(u_{i})$$
$$\psi_{C}^{-1}(u_{i}) = (u_{i}^{-\rho} - 1)$$

$$\begin{split} h_{\rho}^{F}(\mathbf{u}) &= (1 - e^{-\rho})^{1-d} \prod_{j=1}^{d} \{1 - e^{-\rho u_{j}}\} \\ P_{d,\frac{1}{\rho}}^{G}(t^{\frac{1}{\rho}}) &= \sum_{k=1}^{d} a_{dk}^{G}(\frac{1}{\rho})(t^{\frac{1}{\rho}})^{k} \\ a_{dk}^{G}(\frac{1}{\rho}) &= \frac{d!}{k!} \sum_{i=1}^{k} \binom{k}{i} \binom{i}{d} (-1)^{d-i}, \ k \in 1, ..., d \\ t_{\rho}^{G}(\mathbf{u}) &= \sum_{i=1}^{d} \psi_{G}^{-1}(u_{i}) \\ \psi_{G}^{-1}(u_{i}) &= (-\log u_{i})^{\rho} \end{split}$$