Violations of Uncovered Interest Rate Parity and International Exchange Rate Dependences

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   - Covariance Regression
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   - Tail Dependence

4 Conclusions
Impact of the Carry Trade on Japanese Yen

“Borrowing yen to buy higher-yielding assets hasn’t been this popular since the global financial crisis. That’s a rare piece of good news for the Bank of Japan’s bid to achieve 2 percent inflation.”
Bloomberg (March 16, 2015)

“The BOJ’s lending facility promotes yen carry trades, and that’s contributing to yen declines. The trend for a weak yen versus the dollar is likely to continue.”
Takashi Shiono, Economist at Credit Suisse, Tokyo (March 16, 2015)
Impact of the Carry Trade on Worldwide Central Banks

“...Add a resurgent carry trade to the list of things keeping Reserve Bank Governor Glenn Stevens from getting a weaker Aussie dollar...”
Bloomberg (April 19, 2015)

“...Because the kiwi has been put on a pedestal as the carry trade of choice over the last six months, those comments (by the Reserve Bank of New Zealand Assistant Governor John McDermott) have had a big impact this morning...”
A. Myers, Strategist at Credit Agricole (Apr 23, 2015)

“What essentially has happened is the Bank of Thailand has joined the currency war... The central bank has reduced the appeal of the baht from the carry perspective.”
J. Cavenagh, Strategist at Westpac Banking Corp (May 18, 2015)
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Covered Interest Rate Parity

This relation states that the price of a forward rate can be expressed as follows:

\[ F_t^T = e^{(r_t - r_f)(T-t)} S_t \]  

(1)

where \( F_t^T \) and \( S_t \) denote respectively the forward and the spot prices at time \( t \).
Uncovered Interest Rate Parity

If we assume the forward price is a martingale under the risk neutral probability $\mathbb{Q}$, then the fair value of the forward contract at time $t$ equals:

$$E_{\mathbb{Q}}[S_T|\mathcal{F}_t] = F_t^T$$ (2)

where $\mathcal{F}_t$ is the filtration associated to the stochastic process $S_t$. 
Replacing the expression (2) in the relation (1) leads to the UIP equation:

\[ E_Q \left[ \frac{S_T}{S_t} \right] = \frac{F^T_t}{S_t} = e^{(r_t - r^*_t)(T - t)} \] (3)
Uncovered Interest Rate Parity Puzzle

- Well-documented empirical finding that high interest rate currencies tend to appreciate relative to low interest rate currencies

- violating Uncovered Interest rate Parity (UIP)
What is the Carry Trade?

- A simple investment strategy:
  - sell a basket of low interest rate currencies
    (e.g. Japanese yen, Swiss franc,...)
  - buy a basket of high interest rate currencies
    (e.g. Australian dollar, New Zealand dollar,...).

- Numerous empirical studies -
  Hansen and Hodrick 1980; Fama 1984; Engel 1984;
  Lustig and Verdelhan 2007; Brunnermeier, Nagel, and
  Pedersen 2008; Menkhoff et al. 2012

  - have previously demonstrated that investors can actually
    earn profits on average using this strategy.
Cumulative monthly log returns of long and short baskets, which combine to make the carry portfolio.

**Buy**: AUD, NZD, NOK, GBP, CAD.

**Sell**: JPY, CHF, EUR.
Monthly return distribution for carry trade portfolio using 8 Currencies.
Carry Trade Strategy

- Implemented by numerous hedge funds in large size and with high leverage, see Galati, Heath, and McGuire 2007; Fong 2013.

- Also systematic indices sold by investment banks, e.g. SGI FX G10 Carry Trade, DB G10 Currency Harvest or the Barclays Optimized Currency Carry ETN...

However, detection of strategy inflows is not obvious

- as pointed out by Galati, Heath, and McGuire 2007

- generally only verified for downside markets and unwinding periods of the speculative positions.
Carry Trade Strategy

We postulate that the levered construction, as well as the unwinding of the carry trade portfolios, should influence:

- the individual dynamic of the exchange rates
- more importantly the currency commonalities and thus their dependence structure
- the volumes traded on these currencies and more specifically the speculative volume executed in the market
- finally, the dependence between individual speculative volumes
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   • Speculative Volume (SPEC)
   • Mean Regression
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   • Tail Dependence

4 Conclusions
Speculative Volume and Asset Dependences

- Another strand of the literature has shown a link between the asset price variance and the volume traded on the same asset, see He and Velu 2014; Hasbrouck and Seppi 2001; Ané and Geman 2000; Tauchen and Pitts 1983; Clark 1973.

- The Common factor MDH Model (Hasbrouck and Seppi 2001) as well as the multivariate version (He and Velu 2014) are built around the following assumptions:
Speculative Volume and Asset Dependences

The price of asset $i$:

$$P_i = \frac{1}{J} \sum_{j=1}^{J} P_{ij}^*$$  \hspace{1cm} (4)

where $P_{ij}^*$ denotes investor $j$’s forecast for the price of asset $i$.

Furthermore, the volume traded for each asset $i$ can be written as

$$V_i = \frac{c}{2} \sum_{j=1}^{J} \| \Delta P_{ij}^* - \Delta P_i \|$$  \hspace{1cm} (5)

Where $\Delta P_i$ is the market price change, which is assumed to be

$$\Delta P_i = \frac{1}{J} \sum_{j=1}^{J} \Delta P_{ij}^*$$  \hspace{1cm} (6)

Finally, they assume that trader $j$’s incremental price forecast for asset $i$ can be expressed as a combination of asset-specific and cross-asset components

$$\Delta P_{ij}^* = \nu_i^l + \psi_{ij}^l + \nu_i^c + \psi_{ij}^c .$$  \hspace{1cm} (7)
Speculative Volume and Asset Dependences

**Our contributions:**

- Firstly, we show that a large part of the currencies’ speculative volumes are driven by the carry trade (and potentially other well known speculative strategies, e.g. commodity driven).

- We show that speculative volumes have more explanatory power than price index changes for covariance and tail dependences among currencies (Multivariate MDH model).
Speculative Volume and Asset Dependences

Our contributions:

- We propose a dynamic covariance model taking into consideration the stylized facts.
- We demonstrate that not only the downward extremal dependence among currencies, but also the upward equivalent are impacted by speculator behaviour.
- Also, we extended our model to 34 currencies (mixing developed and developing countries).
Data and Carry Trade Portfolio Construction

- 8 currencies as studied in Brunnermeier, Nagel, and Pedersen 2008
- High Interest Rate Basket: Australia (AUD), Canada (CAD), New Zealand (NZD), Norway (NOK), United Kingdom (GBP)
- Low Interest Rate Basket: Japan (JPY), Switzerland (CHF), Euro (EUR)
Data and Carry Trade Portfolio Construction

Daily settlement prices:
- spot exchange rate
- associated 1 month forward contract
- 02/01/1989 - 29/01/2014

Weekly speculative open interest ratio:
- $\text{SPEC} = (\text{NNCFP} / \text{Open Interest})$
- 20/06/2006 - 29/01/2014
- Norway (NOK) SPEC unavailable
High Interest Rate Basket: Spot Prices

Spot prices normalised by spot price on 02/01/1999.
Spot prices normalised by spot price on 02/01/1999.
Price Factors: $\text{DOL} + HML_{\text{FX}}$

**Left Subplot:** Loadings of the 1st PC of Currency Returns.

**Right Subplot:** Loadings of the 2nd PC of Currency Returns.
Price Factors: $DOL + HML_{FX}$

Suggested by Lustig and Verdelhan 2007:

- $DOL$: average excess return of all currencies against the dollar
- $HML_{FX}$: High interest rate basket return - Low interest rate basket return
High Interest Rate Basket: SPEC

SPEC of High Interest Rate Basket Currencies

SPEC = (NNCFP / Open Interest)

SPEC = NNCFP / OI
Low Interest Rate Basket: SPEC

SPEC = \frac{\text{NNCFP}}{\text{Open Interest}}
Speculative Open Interest (SPEC)

Loadings of the 1st PC of Speculative volume ratio (SPEC).
Currency Returns and SPEC

How does SPECulative volume affect currency returns:

✓ mean
  • covariance
  • tail dependence
### Mean regression: Individual currency returns $\sim$ SPEC

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<th>CHF</th>
<th>EUR</th>
<th>GBP</th>
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<td>(0.005)</td>
<td>(0.504)</td>
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<td>-0.001</td>
<td>-0.004</td>
<td>-0.001</td>
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<tr>
<td></td>
<td>(0.560)</td>
<td>(0.088)</td>
<td>(0.590)</td>
<td>(0.804)</td>
<td>(0.271)</td>
<td>(0.044)</td>
<td>(0.712)</td>
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</table>

\[ R^2(\text{DOL, HML}_{\text{FX}}) = 0.916, \quad R^2(\text{DOL, HML}_{\text{FX}}, \text{SPEC}) = 0.920 \]

Individual currency returns $\sim DOL + HML_{\text{FX}} + \text{SPEC}$.

Numbers in parentheses show Newey and West (1987) HAC p-values.
Currency Returns and SPEC

How does SPECulative volume affect currency returns:

- mean
- covariance
- tail dependence
Covariance regression: Multivariate Currency Returns $\sim$ SPEC

- Firstly, we de-trend the currency returns using the mean regression (as before) to get residuals $e_i = y_i - \beta x_i$,

$$x_i = [DOL(i), HML_{FX}(i)]^T$$

or

$$x_i = [DOL(i), HML_{FX}(i), SPEC(i)]^T$$

or

$$x_i = [DOL(i), HML_{FX}(i), SPEC(i), SPEC(i) \ast SPEC(i)]^T.$$  

- We model the covariance matrix for $e_i$, conditional on $x_i$ given by,

$$\mathbb{E}[e_i e_i^T | x_i] = Bx_i x_i^T B^T + \Psi$$  \hspace{1cm} (8)

- Convenient to use the following random-effects representation:

$$e_i = \gamma_i \times Bx_i + \epsilon_i$$

$$\mathbb{E}[\epsilon_i] = 0, \quad \text{Cov}(\epsilon_i) = \Psi$$

$$\mathbb{E}[\gamma_i] = 0, \quad \text{Var}[\gamma_i] = 1, \quad \mathbb{E}[\gamma_i \times \epsilon_i] = 0$$

$$\mathbb{E}[e_i e_i^T | x_i] = \mathbb{E}[\gamma_i^2 Bx_i x_i^T B^T + \gamma_i (Bx_i \epsilon_i^T + \epsilon x_i^T B^T) + \epsilon_i \epsilon_i^T]$$

$$= Bx_i x_i^T B^T + \Psi$$  \hspace{1cm} (9)
Covariance regression: Multivariate Currency Returns $\sim$ SPEC

Estimate coefficients via EM:

- Iteratively maximising the complete data log-likelihood

$$l(B, \Psi) = \log p(E|B, \Psi, X, \gamma),$$
which is obtained from the multivariate normal density given by

$$-2l(B, \Psi) = np \log(2\pi) + n \log|\Psi| + \sum_{i=1}^{n} (e_i - \gamma_i Bx_i)^T \Psi^{-1} (e_i - \gamma_i Bx_i).$$ (10)

- We note that the conditional distribution of the random effects given the data and covariates is then conveniently given by a normal distribution in each element according to $\{\gamma_i|E, X, \Psi, B\} = \mathcal{N}(m_i, \nu_i)$ with mean $m_i = \nu_i (e_i^T \Psi^{-1} Bx_i)$ and variance $\nu_i = (1 + x_i^T B^T \Psi^{-1} Bx_i)^{-1}$ see Hoff and Niu 2011.
Covariance regression: Multivariate Currency Returns $\sim$ SPEC

EM algorithm: Expectation Step

- Initialize the parameter matrices, $\hat{\Psi}$ and $\hat{B}$.
- Calculate the conditional estimators:

$$m_i = \mathbb{E}\left[\Gamma_i | \hat{\Psi}, \hat{B}, e_i\right]$$

$$v_i = \text{Var}\left[\Gamma_i | \hat{\Psi}, \hat{B}, e_i\right]$$

(11)

- Construct new matrices $\tilde{X}$ and $\tilde{E}$ based on the data $y_{1:n}$ and covariates $x_{1:n}$.

  $\tilde{E}$ is the $2n \times 1$ matrix given by $(E^T, 0 \times E^T)^T$

  $\tilde{X}$ is a $2n \times d$ matrix with $i$-th row given by $m_i x_i$ and whose $(n + i)$-th is $\sqrt{v_i}x_i$. 
Covariance regression: Multivariate Currency Returns $\sim$ SPEC

EM algorithm: Maximisation Step

- Evaluate the updated model parameters via the following least squares solutions for updated $\hat{\Psi}$ and $\hat{B}$ according to

\[
\hat{B} = \tilde{E}^T \tilde{X} \left( \tilde{X}^T \tilde{X} \right)^{-1} \\
\hat{\Psi} = \frac{1}{n} \left( \tilde{E} - \tilde{X} \hat{B} \right)^T \left( \tilde{E} - \tilde{X} \hat{B} \right)
\]

- repeat the above procedure until convergence.
Covariance regression: Multivariate Currency Returns \( \sim \) SPEC

Non-Baseline Variance \( \% = 100 \times \frac{\text{trace}(BX_{(0.5)}X_{(0.5)}^T B^T)}{\text{trace}(BX_{(0.5)}X_{(0.5)}^T B^T) + \text{trace}(\Psi)} \)

- The proportion of covariation in the covariance regression attributed to the factors relative to the total second order explanatory power of the covariance regression on each 125 week sliding window.
- This measure focuses on the covariance explained when \( X \) takes its median value, denoted \( X_{(0.5)} \).
Covariance regression: Multivariate Currency Returns $\sim$ SPEC

**Percentage of Variance Accounted for by Non-Baseline Variance in High and Low Interest Rate Baskets**

- **High IR Basket:** DOL + HML
- **High IR Basket:** DOL + HML + SPEC + SPEC$^2$
- **Low IR Basket:** DOL + HML
- **Low IR Basket:** DOL + HML + SPEC + SPEC$^2$

**CHF Pegged to Euro**

**EZ Debt Crisis**

$DOL + HML_{FX}$ vs $DOL + HML_{FX} + SPEC + SPEC \times SPEC$.

125 week lookback periods.
Currency Returns and SPEC

How does SPECulative volume affect currency returns:

- mean
- covariance

✓ tail dependence
Modelling the Multivariate Exchange Rate Returns of the Baskets

We consider the two stage estimation procedure known as the inference function for margins (IFM) technique as studied in Joe and Xu 1996.

Stage 1: estimation of suitable heavy tailed marginal models

Stage 2: followed by estimation of the dependence component of the multivariate model
We initially considered LogNormal margins:

Percentage of rejections using the Kolmogorov-Smirnov test at 5% significance level for log normal margins.
Modelling the Marginals - Log Generalized Gamma

Since a number of the currencies appearing in the high interest rate basket failed K-S tests for a significant portion of days:

- We considered a more flexible three parameter model:

  Generalized-Gamma distribution

  \[ f_X(x; k, \alpha, \beta) = \frac{\beta}{\Gamma(k)} \frac{x^{\beta k - 1}}{\alpha^{\beta k}} \exp \left( - \left( \frac{x}{\alpha} \right)^\beta \right) \]  

  \[ \Rightarrow \text{ Log-Generalized-Gamma distribution (l.g.g.d.)} \]

  \[ f_Y(y; k, u, b) = \frac{1}{b \Gamma(k)} \exp \left[ k \left( \frac{y - u}{b} \right) - \exp \left( \frac{y - u}{b} \right) \right] \]  

  with \( u = \log(\alpha), \ b = \beta^{-1}, \text{supp}(y) \in \mathbb{R}. \)
We model the dependence between log exchange rate returns using a Clayton-Frank-Gumbel mixture copula:

- asymmetric flexibility in the tails
- automatic model selection
Mixture Copula

**Definition (Mixture Copula)**

*A mixture copula is a linear weighted combination of copulae of the form:*

\[
C_M(u; \Theta) = \sum_{i=1}^{N} w_i C_i(u; \theta_i) \tag{15}
\]

*where \( 0 \leq w_i \leq 1 \ \forall i = 1, ..., N \)*

*and \( \sum_{i=1}^{N} w_i = 1 \)
Mixture Copula

Thus we can combine:

- a copula with lower tail dependence
- a copula with positive or negative dependence
- a copula with upper tail dependence

...to produce a more flexible copula capable of modelling the multivariate log returns of forward exchange rates of a basket of currencies.
Multivariate Tail Dependence

Using the fitted mixture of copula functions:

- we can extract a measure of the multivariate tail dependence within each basket

- see De Luca and Rivieccio 2012.
### Multivariate Tail Dependence - Upper

**Definition (Generalized Upper Tail Dependence Coefficient)**

Let $X = (X_1, ..., X_d)^T$ be a $d$ dimensional random vector with marginal distribution functions $F_1, ..., F_d$. The coefficient of upper tail dependence is defined as:

\[
\lambda_{u_{1,...,h|h+1,...,d}} = \lim_{\nu \to 1^-} P (X_1 > F^{-1}(\nu), ..., X_h > F^{-1}(\nu) | X_{h+1} > F^{-1}(\nu), ..., X_d > F^{-1}(\nu))
\]

\[
= \lim_{t \to 0^+} \frac{\sum_{i=1}^{d} \left( \binom{d}{d-i} i (-1)^i \left[ \psi^{-1'}(it) \right] \right)}{\sum_{i=1}^{d-h} \left( \binom{d-h}{d-h-i} i (-1)^i \left[ \psi^{-1'}(it) \right] \right)}
\]

(16)
Multivariate Tail Dependence - Lower

Definition (Generalized Lower Tail Dependence Coefficient)

Let \( X = (X_1, ..., X_d)^T \) be a \( d \) dimensional random vector with marginal distribution functions \( F_1, ..., F_d \). The coefficient of lower tail dependence is defined as:

\[
\lambda_{1,\ldots,h|h+1,\ldots,d} = \lim_{\nu \to 0^+} P \left( X_1 < F^{-1}(\nu), \ldots, X_h < F^{-1}(\nu) \mid X_{h+1} < F^{-1}(\nu), \ldots, X_d < F^{-1}(\nu) \right)
\]

\[
= \lim_{t \to \infty} \frac{d}{d - h} \frac{\psi^{-1}'((d - h)t)}{\psi^{-1}'(dt)}
\]

(17)
Tail Dependence - High Interest Rate Basket

Comparison of Volatility Index (VIX) with upper and lower tail dependence of the high interest rate basket.
### High interest rate respective tail dependences

\[
\tilde{\lambda}_I^H = DOL + HML_{FX} + \sigma_{DOL} + \sigma_{HML_{FX}} + \sigma_{DOL,HML_{FX}} + AUD + CAD + CHF + EUR + GBP + JPY + NZD + CROSS + R^2
\]

Numbers in parentheses show Newey and West (1987) HAC p-values.

\[
\tilde{\lambda}_u^H = -0.023 + 0.030 -2.343 + 23.177 -13.838 -0.129 + 0.126 -0.023 + 0.154 + 0.046 -0.023 + 0.008 \times 27.1%
\]

\[
\tilde{\lambda}_u^H = -0.162 -0.032 + 0.625 + 16.159 -201.897 -0.173 + 0.281 -0.014 + 0.232 + 0.133 -0.053 -0.020 \times 39.4%
\]

**Summary:**

- \( DOL \sim SPEC \)
- \( HML_{FX} \sim SPEC \)
- \( \sigma_{DOL} \sim SPEC \)
- \( \sigma_{HML_{FX}} \sim SPEC \)
- \( \sigma_{DOL,HML_{FX}} \sim SPEC \)
- \( AUD \sim SPEC \)
- \( CAD \sim SPEC \)
- \( CHF \sim SPEC \)
- \( EUR \sim SPEC \)
- \( GBP \sim SPEC \)
- \( JPY \sim SPEC \)
- \( NZD \sim SPEC \)
- \( CROSS \sim SPEC \)

**Conclusions:**

- High interest rate respective tail dependences ~ \( DOL + HML_{FX} + DOL_{VOL} + HML_{VOL} + DOL \times HML_{FX} + SPEC + SPEC \times SPEC \).
- Numbers in parentheses show Newey and West (1987) HAC p-values.
Comparison of Volatility Index (VIX) with upper and lower tail dependence of the low interest rate basket.
Low interest rate respective tail dependences $\sim DOL + HML_{FX} + DOL_{VOL} + HML_{VOL} + DOL \times HML_{FX} + SPEC + SPEC \times SPEC$. Numbers in parentheses show Newey and West (1987) HAC p-values.
6-month rolling upper tail dependence of low interest rate developed countries (namely JPY, CHF, EUR) compared to net open position on the Swiss franc future contract.
Outline

1 Introduction
2 UIP and the Carry Trade
   - Covered Interest Rate Parity
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3 Speculative Volume and Currency Returns
   - Literature and Our Contributions
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4 Conclusions
Conclusions

- A large part of the currencies’ speculative volumes is driven by the carry trade.

- Speculative volumes have more explanatory power than price index changes for covariance and tail dependences among currencies.

- Downward and upward extremal dependences among currencies are impacted by speculator behaviour.
Conclusions

- During crisis periods the high interest rate currencies tend to display very significant upper tail dependence.

- High interest rate and low interest rate currency baskets can display periods during which the tail dependence gets inverted, showing construction periods of carry positions.

- Tail dependence can be used to measure the construction and unwinding of speculative positions in assets for which we don’t have speculative volume data!
Questions?


References III


Profile likelihood plots for C-F-G mixture model.
Profile likelihood plots for C-F-G mixture model.
Clayton-Frank-Gumbel Mixture Copula

\[ C_{\rho_C, \rho_F, \rho_G}^{CFG}(u) = \lambda_C(C_{\rho_C}^{\lambda}(u)) + \lambda_F(C_{\rho_F}^{\lambda}(u)) + \lambda_G(C_{\rho_G}^{\lambda}(u)) \]

\[ = \lambda_C \times \left( \sum_{i=1}^{d} u_i^{-\rho} - d + 1 \right)^{-\frac{1}{\rho}} \]

\[ + \lambda_F \times -\frac{1}{\rho} \ln \left( 1 + \frac{\prod_{i=1}^{d} (e^{-\rho u_i} - 1)}{(e^{-\rho} - 1)^{d-1}} \right) \quad (18) \]

\[ + \lambda_G \times e^{-\left( \sum_{i=1}^{d} (-\log u_i)^\rho \right)^{\frac{1}{\rho}}} \]
Clayton-Frank-Gumbel Mixture Copula Density

\[ c_{\rho_C, \rho_F, \rho_G}^{CFG}(u) = \lambda_C(c_{\rho_C}^C(u)) + \lambda_F(c_{\rho_F}^F(u)) + \lambda_G(c_{\rho_G}^G(u)) \]

\[ = \lambda_C \times \prod_{k=0}^{d-1} (\rho k + 1) \left( \prod_{i=1}^{d} u_i \right)^{-(1+\rho)} (1 + t_{\rho}^C(u))^{(-d+\frac{1}{\rho})} \]

\[ + \lambda_F \times \left( \frac{\rho}{1 - e^{-\rho}} \right)^{d-1} Li_{-(d-1)} \left\{ h_{\rho}^F(u) \right\} \frac{e^{\left(-\rho \sum_{j=1}^{d} u_j\right)}}{h_{\rho}^F(u)} \]

\[ + \lambda_G \times \rho^d e^{\left(-t_{\rho}(u)^{\frac{1}{\rho}}\right)} \prod_{i=i}^{d} (-\log u_i)^{\rho-1} \]

\[ P_{d, \frac{1}{\rho}}^{G}(t_{\rho}^G(u)^{\frac{1}{\rho}}) \]
where

\[ t^C_\rho(u) = \sum_{i=1}^{d} \psi^{-1}_C(u_i) \]
\[ \psi^{-1}_C(u_i) = (u_i^{-\rho} - 1) \]

\[ h^F_\rho(u) = (1 - e^{-\rho})^{1-d} \prod_{j=1}^{d} \{1 - e^{-\rho u_j}\} \]

\[ P^G_{d, \frac{1}{\rho}}(t^{\frac{1}{\rho}}) = \sum_{k=1}^{d} a^G_{dk}(\frac{1}{\rho})(t^{\frac{1}{\rho}})^k \]
\[ a^G_{dk}(\frac{1}{\rho}) = \frac{d!}{k!} \sum_{i=1}^{k} \binom{k}{i} \binom{i}{d}(\frac{1}{\rho})^{d-i}, \quad k \in 1, ..., d \]

\[ t^G_\rho(u) = \sum_{i=1}^{d} \psi^{-1}_G(u_i) \]
\[ \psi^{-1}_G(u_i) = (-\log u_i)^\rho \]