Loss Reserve

Lucas (Huijie) Tian*

* Supervisor: Gareth W. Peters

Department of Statistical Science, University College London, UK

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Table of contents

Table of contents

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- The amount the insure is obligated to pay is known as the claim/loss amount.
- The payment that makes up the claims are known as the the claims/loss payment

Life and General Insurance

In general, insurance products can be sorted into **Two** classes: General insurance and Life Insurance. The main reason for this is:

- The life and general products are modelled in a different way, due to their nature. for example, the terms of contracts, types of claim, risk drivers,etc.
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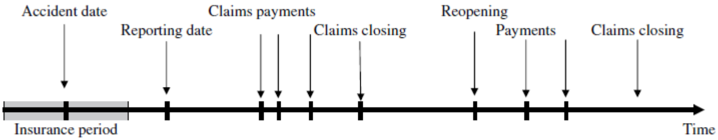
we could focus on the general insurance claim reserving problem.

Line of Business

The General Insurance operates on Following LoB:

- Motor Insurance
- Property Insurance
- Liability Insurance
- Accident Insurance
- etc.

Process Time Line for Insurance Product



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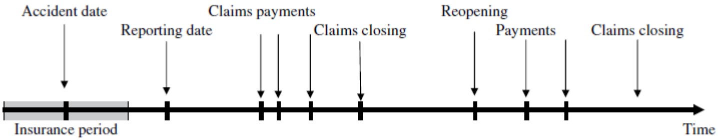

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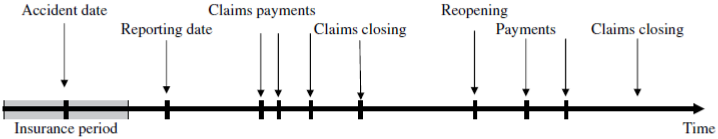

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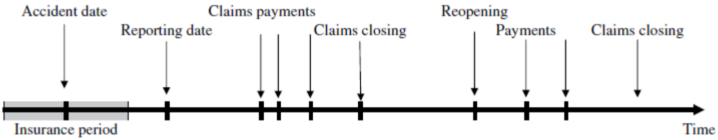

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- The claim may settled with take several payment after being reported, AKA, Settlement Delay
- Some closed claim is reported due to unexpected new developments, AKA, Reporting Delay

Principle

In order to have a consistent financial statement, it is important that the accident date and the premium accounting principle are compatible via the exposure pattern, which means the claim occurred at each accident year must match the premium earned that year.

Reserves

In term of the reserve, we need to build the reserve for future exposures (UPR, unearned premium reserve) and for the past exposures. In general, there are two kinds of claim reserve for the past exposures:

- *IBNR(Incured but Not Report)*
- *RBNS(Report but Not Settled)*

Why we need Reserves:

The reserves are set up for various reason which depend on certain circumstances, the following are some typically example: (faculty and institution of actuaries, claim reserve manual 09/1997)

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- Putting a value on the net worth of an insurer, particularly for purposes of sales or acquisitions;
- Commutations and reinsurance to close: that is, putting a financial value on the run-off of a portfolio of insurance business.

Mathematical Framework

In this section, the mathematical framework will be introduced (follow Arjas 1989).

- Assume there are N Claims within a fixed time period
- $T_1, \dots T_N$ denotes the reporting dates with $T_i \leq T_{i+1}$
- For the i-th claim, Let $T_i = T_{i,0} \leq T_{i,1} \leq \dots \leq T_{i,j} \dots \leq T_{i,N_i}$ denotes a sequence of dates at with there are some action on claim i is observed.
  T_{i,N_i} denotes the final settlement of the claim.
- At time $T_{i,j}$, we may have a payment, a new estimation by claim adjusted, or other information related to the claim

$$X_{i,j} = \begin{cases} \text{payment at time} T_{i,j} \text{ for claim } i \\ 0, \text{no payment at time} T_{i,j} \end{cases}$$

$$I_{i,j} = \begin{cases} \text{new information availibale at time} T_{i,j} \text{ for claim } i \\ \emptyset, \text{no new information at time} T_{i,j} \end{cases}$$

with above framework, the reserving problem can be split into different process:

- **Payment Process of Claim i** given by $(T_{i,j}, X_{i,j})_{j \geq 0}$
- **Information Process of Claim i** given by $(I_{i,j}, T_{i,j})_{j \geq 0}$
- **Settlement Process of Claim i** given by $(T_{i,j}, X_{i,j}, I_{i,j})_{j \geq 0}$

Cumulative Payment Process

$C_i(t) = \sum_{j \in \{k; T_{i,k} \leq t\}} X_{i,j}$

$C_i(\infty) = C_i(T_{i,N_i}) = \sum_{j=0}^{\infty} X_{i,j}$

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Then the Total outstanding claim payment for future liabilities of claim i at time t is a random variable given by: (need to be predicted)
$R_i(t) = C_i(\infty) - C_i(t) = \sum_{j \in \{k; T_{i,k} > t\}} X_{i,j}$

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Aggregate Process

For *all claims* we can defined: $C(t) = \sum_{i=1}^{N} C_i(t)$, the all payments up to time t for all N claims $R(t) = \sum_{i=1}^{N} R_i(t)$, the amount of outstanding claims payments at time t for these N claims.

The outstanding loss liabilities are studied in **Claim Development Triangles**,

Run-off Triangle

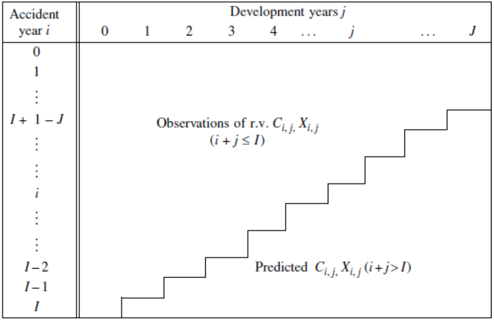
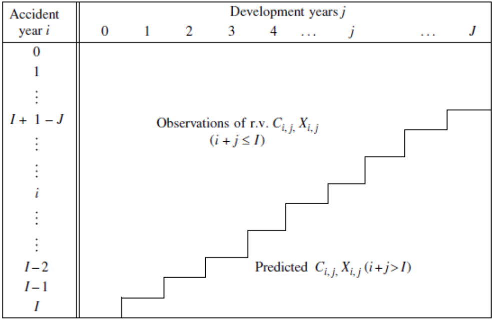


Figure 1.2 Claims development triangle



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Vertical axis: accident year (i)
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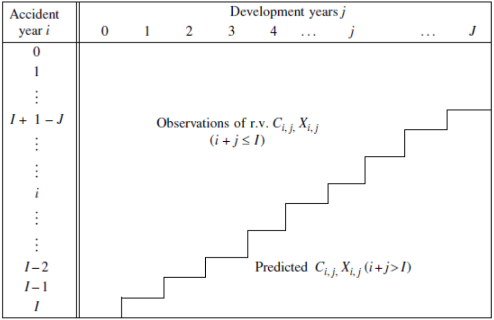
The claim development triangle can be divided into two parts:

Run-off Triangle



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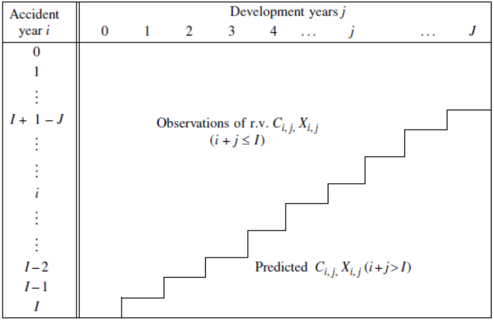

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- Upper Triangle: the Observation and defined by:
  $\mathbb{D}_I = \{X_{i,j}; i+j \leq I, 0 \leq j \leq J\}$
- Lower Triangle: the Prediction and defined by:
  $\mathbb{D}_I^c = \{X_{i,j}; i+j > I, i \leq I, j \leq J\}$

Table of contents

Chain Ladder Method

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- Operates on aggregate loss data (sums of individual paid claims).
- The name given to this method presumably arises from the ladder-like operations which are chained over the development years
- From theoretical point of view this naturally gives rise to a compound Poisson distribution

Assumption

- *Payments from each accident year will develop in the same way.*
- *Changes in the rate at which claims emerge can only be incorporated by ”hand adjustment” of the development factors.*
- *Weighted average past inflation will be repeated in the future*
- *claims inflation is one of the influences swept up within the projection factors,*

Definition: Development Factor

The development factors measure the proportionate increases in the known cumulative payments from one development year to the next, which is a volume-weighted average ratio that are used to project future claims in each accident year.

$$\hat{f}_j = \frac{\sum_{i=0}^{I-j-1} C_{i,j+1}}{\sum_{i=1}^{I-J-1} C_{i,j}} = \sum_{i=0}^{I-j-1} \frac{C_{i,j}}{\sum_{k=0}^{I-j-1} C_{C_{k,j}}} \frac{C_{i,j+1}}{C_{i,j}}$$

Then we have:

$$\hat{C}_{i,j+1} = \hat{f}_j C_{i,j}$$

Property of Development Factor

- the development factor is an unbiased estimator
- the development factor are uncorrelated with each other

Chain Ladder Method; continued

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- Bootstrap Chain Ladder (England and Verrall, 2007)

Bornhuetter-Ferguson (BF) method is another well known method in both theory and practice which proposed by Bornhutter and Ferguson (1972) in his famous article called ' The actuary and IBNR'.

Definition: Loss Ratio

The loss ratio is the ratio of the ultimate incurred claims to the premium income for the polices in the accident year. It is useful for think about likely outstanding claims

Assume that there have not been extreme events that would make historical loss ratios unhelpful for the current claims. The BF method uses the historical loss ratio to get an initial estimate of the ultimate loss then factors in the development of the claims incurred so far.

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The future claims development can be expressed as: Future claims development= Premium $\times$ Estimated Loss Ratio $\times$ $(1 - 1/f)$

- B-F method could be viewed as using a Bayesian approach.
- Using B-F method, each years revised ultimate loss can be expressed as a combination of the chain ladder ultimate loss (CL) and the independent loss ratio of ultimate loss (LR).
- Using a credibility factor (Z), this can be expressed as:

$$\text{B-F revised ultimate loss} = ZCL + (1 - Z)LR$$

- $$\text{B-F revised ultimate loss } = A + (1 - 1/f) \times LR$$

  A is the actual claims developed to date figure (i.e. the figure from the latest lead diagonal in the run-off triangle.)
  Z = A/CL is just the inverse of the ultimate development factor (f) for the year.

Table of contents

Motivation

Many research now starts working on individual claim data, i.e Arjas(1989) and Norberg (1993,1999).
Double Chain Ladder Model is a micro-level stochastic model using the aggregate data instead of using historical individual data. and it is a basic extension of Verrall's work (2010).

- it overcomes the ”ad hoc” feature of CLM
- it explicitly acknowledge that data are compound poisson process.
- cape with IBNR and RBNS delay in a very easy way
- the full model describe the cash flow of outstanding RBNS liability.

Double Chain Ladder Method: Assumption

Let $X_{i,j}$ denote the observed total payment for claims incurred at time i and paid at $i+j$, then $N_{i,j}$ denotes the observed total number of claims incurred at i and reported at $i+j$.

Now define $N_{i,j}^{paid}$ be the number of payments originating from the $N_{i,j}$ incurred in year i and settled in $i+j$, and d denote the maximum periods of delay ($d \leq m-1$)

Assumption

- Variables in different accident year i or development year j are independent
- The individual claims are settled with a single payment or maybe "zero payment", and let $Y_{i,j}^{(k)}$ denote the individual settled payments which arise from $N_{i,j}^{paid}$. ($k = 1, \ldots, N_{i,j}^{paid}$). Then we have $X_{i,j} = \sum_{k=0}^{N_{i,j}^{paid}} Y_{i,j}^{(k)}$.

Double Chain Ladder Method: Assumption

Continued

- $Y_{i,j}^{(k)}$ are mutually independent with mean μ_i and the variance σ_i such that $E[Y_{i,j}^{(k)}] = \mu\gamma_i = \mu_i$ and $Var[Y_{i,j}^{(k)}] = \sigma^2\gamma_i^2 = \gamma_i^2$ with μ and σ^2 being the mean and variance factors and γ_i be the inflation in accident year i
- $Y_{i,j}^{(k)}$ are independent of the counts $N_{i,j}$ and also the RBNS and IBNR delays.
- The counts $N_{i,j}$ are independent random variable from a Poisson Distribution with multiplicative Parametrization: $E[N_{i,j}] = \alpha_i\beta_j$ and $\sum_{j=0}^{m-1} \beta_j = 1$

Double Chain Ladder Method: Assumption

Assumption

Let d denote the maximum period of delay, then:
$N^{paid}_{i,j} = \sum_{l=0}^{min(j,d)} N^{paid}_{i,j-l,l}$, where $N^{paid}_{i,j-l,l}$ means the number of payments originating from $N_{i,j}$ reported claims and paid in $i+j$
assume that $(N^{paid}_{i,j,0}, \ldots, N^{paid}_{i,j,d}) \sim multi(N_{i,j}; p_0, \ldots, p_d)$ with $\mathbf{p} = (p_0, \ldots, p_d)$ being the delay probabilities

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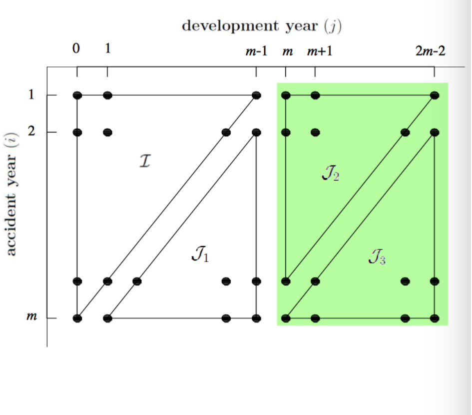
Since $N^{paid}_{i,j} = \sum_{l=0}^{min(j,d)} N^{paid}_{i,j-l,l}$, we have:
$E[N^{paid}_{i,j}|N_m] = \sum_{l=0}^{min(j,d)} N_{i,j-l}p_l$.
Since $X_{i,j} = sum_{k=0}^{N^{paid}_{i,j}} Y^{(k)}_{i,j}$, we have ;
$E[X_{i,j}|N_m] = E[N^{paid}_{i,j}|N_m]E[Y^{(k)}_{i,j}] = \sum_{l=0}^{min(j,d)} N_{i,j-l}p_l\mu\gamma_i$
$E[X_{i,j}] = \alpha_i\mu\gamma_i \sum_{l=0}^{min(j,d)} N_{i,j-l}p_l$

Double Chain Ladder Method: Algorithm



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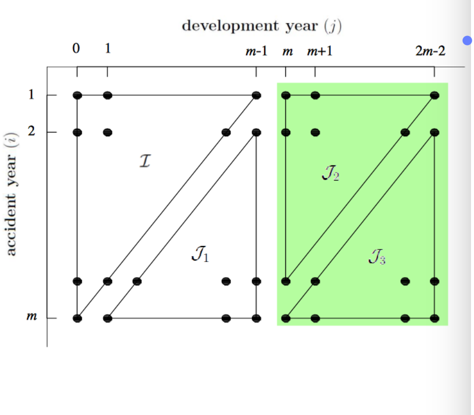

- RBNS Prediction:
  $\hat{X}^{rbns}_{i,j} = \sum_{l=i-m+j}^{min(j,d)} N_{i,j-l}\hat{p}_l\hat{\mu}\hat{\gamma}_i$
  with $(i,j) \in \mathcal{J}_1 \cup \mathcal{J}_2$

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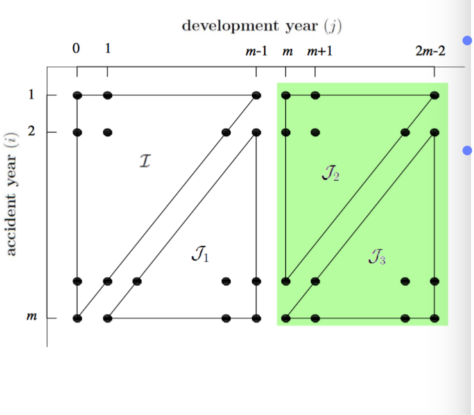

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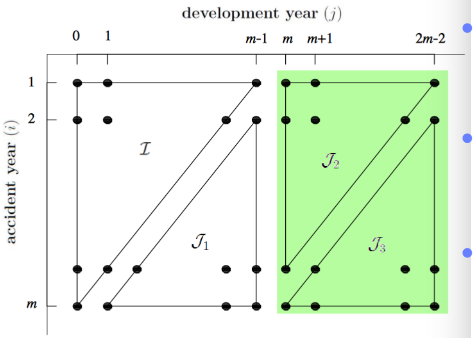

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  with $(i,j) \in \mathcal{J}_1 \cup \mathcal{J}_2 \cup \mathcal{J}_3$
- Tails:
  $\sum_{(i,j)\in\mathcal{J}_2\cup\mathcal{J}_3} \sum_{l=0}^{min(j,d)} N_{i,j-l}\hat{p}_l\hat{\mu}\hat{\gamma}_i$
  with $(i,j) \in \mathcal{J}_3 \cup \mathcal{J}_2$

Table of contents

Raw Data

CTP insurance is compulsory insurance, mandatory for every motor vehicle registered in Australia, privately underwritten by licensed insurers.

It covers the negligent at-fault driver for their liability to compensate third parties for bodily injury caused by them involving the operation of their vehicle.

This includes the driver(s) and passengers in other vehicle(s) involved in the accident, passengers in your vehicle, pedestrians, push-bike and motorcycle riders. It also provides more limited benefits to the at-fault driver themselves.

Our data is based on the CTP insurance from Queensland insurance companies.

The Data contains 115,344 accident records starting from Sept,1994 to Dec,2008. The data is saved into three different files, the Claim Information file, the Claim payment file and the estimate history file.

Table of contents

introduction

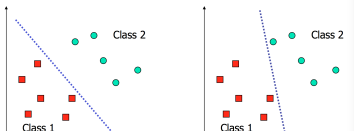
The Support Vector Machine is a new generation of supervised learning algorithm introduced in COLT-92by Boser, Guyon and Vapnik for Binary Pattern Recognition.

It become popular because of its success in handwritten digit recognition.

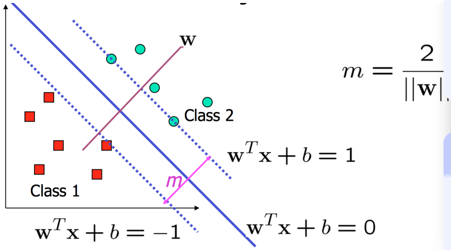
Now, SVM is regarded as an important example of kernel method, one of the key areas in machine learning - that we introduce to insurance and risk management reserving

The SVM try to find the optimal separating hyperplane between two classes by maximizing the margin between the classes' closets points lying on the boundaries which direct bearing on the optimal location of the decision surface and are called Support Vectors.

The multi-class classification can be done using one against one technique by fitting all binary subs-classifiers and finding the correct class by a voting mechanism.



Large-Margin linear Classifier: Linear Separable



notation

Let $\{x_1, x_2, \cdots, x_n\}$ be our data set, and $y_i \in \{-1, 1\}$ be the class label of x_i, Then, the decision boundary should classify all points correctly:

$$y_i(\mathbf{w}^T\mathbf{X_i} + b) \geq 1 \quad \forall i$$

optimal decision boundary

maximize the margin m and this can be formulated as a constrained optimization problem:

Minimum $\frac{1}{2}|\mathbf{w}|^2$
subject to $y_i(\mathbf{w}^T\mathbf{x_i} + b) \geq 1, \forall i$

Constrained Optimization

Suppose we want to: minimize $f(x)$ subject to $g(x) = 0$, A necessary condition for x_0 to be a solution:

$$\begin{cases} \frac{d}{d_x}(f(x) + \alpha g(x))|_{x=x_0} = 0 \\ g(x) = 0 \end{cases}$$

α : the Lagrange multiplier
For multiple constraints $g_i(x) = 0, i = 1, ..., m$, we need a Lagrange multiplier α_i for each of the constraints

$$\begin{cases} \frac{d}{d_x}(f(x) + \sum_{i=1}^{n} \alpha_i g(x)_i)|_{x=x_0} = 0 \\ g_i(x) = 0 \forall i \end{cases}$$

The function is also known as the Lagrangian; we want to set its gradient to 0

Large-Margin linear Classifier:linear Separable

Primal Problem

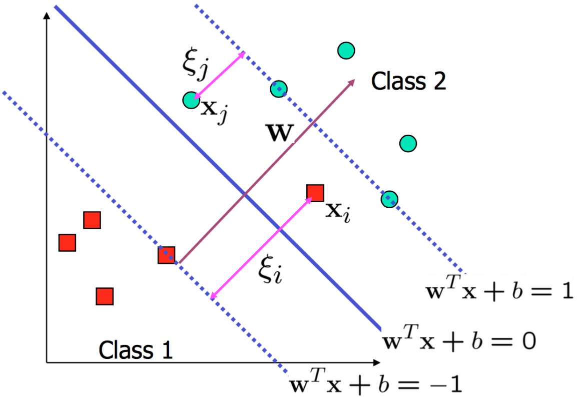
The original boundary problem is minimize $\frac{1}{2}|\mathbf{w}|^2$, subject to
$1 - y_i(\mathbf{w}^T\mathbf{x_i} + b) \leq 0, \text{for} i = 1, \cdots, n$
The Lagrangian is $\mathbb{L} = \frac{1}{2}w^T w + \sum_{i=1}^{n} \alpha_i(1 - y_i(w^T x_i + b))$
setting the gradient of $\mathbb{L}$ with respect to W and b to be zero, we have :
$w + \sum_{i=1}^{n} \alpha_i(-y_i)x_i = 0$ and $\sum_{i=1}^{n} \alpha_i y_i = 0$

Dual Problem

if we substitute $w = \sum_{i=1}^{n} \alpha_i y_i x_i$ into $\mathbb{L}$, we obtain a new objective function in term of α_i only.
$\mathbb{L} = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i\alpha_j y_i y_j \mathbf{x}_i^T\mathbf{x}_j + \sum_{i=1}^{n} \alpha_i$
This is known as dual problem: and it can be formalised as:
max $\quad W(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i\alpha_j y_i y_j \mathbf{x}_i^T\mathbf{x}_j$
subject to : $\alpha_i \geq 0$, $\sum_{i=1}^{n} \alpha_i y_i = 0$.

Large-Margin linear Classifier:Non-linear Separable

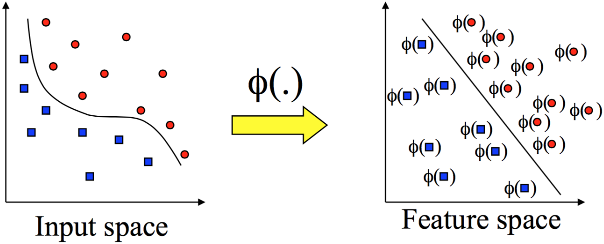
we allow error ξ_i in classification, it is based on the output of the discriminant function $\mathbf{w}^T\mathbf{X} + b$



ξ_i approximates the number of misclassified samples. and this can of optimization problem are called soft margin problem.

Non-linear Decision Boundary: transformation

If the data are non-linear separable, we can transform x_i to a higher dimensional space, called feature space, which means the linear operation in the feature space is equivalent to non-linear operation in input space.



[1]

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[1] the feature space is of higher dimension than the input space in practice

Kernel function

recall the SVM dual optimization problem:
max $W(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j \mathbf{x}_i^T \mathbf{x}_j$
subject to : $\alpha_i \geq 0$, $\sum_{i=1}^{n} \alpha_i y_i = 0$.
The data points only appear as inner product, hence as long as we can calculate the inner product in the feature space, we do not need the mapping explicitly.

Kernel Function

Define the Kernel function K by: $K(x_i, x_j) = \phi(x_i) T \phi(x_j)$
Example of kernel:

- Polynomial kernel with degree d; $K(x, y) = (x^T y + 1)^d$
- Radial basis function kernel with width σ; $K(x, y) = exp(-|x - y|^2)/2\sigma^2$

Table of contents

Ongoing work involves:

- characterising the loss processes for different classes of claims - through feature vector selections and SVM
- Building individual loss process simulation models for each type of claims process identified from the classification of the processes
- Using the build models to simulate loss histories and then aggregating these to form loss triangles, comparing the results on loss triangles to Double chain ladder models
- Identifying which stochastic features can be captured by double chain ladder models but not identified by aggregate claims models like classical chain ladder methods.