Advanced statistical models for cryptocurrency research

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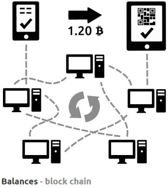
Outline

What is cryptocurrency?

- A decentralised digital currency that uses cryptography
  - to secure its online transactions,
  - to control the creation of additional units (limited to 21 million for say Bitcoins), and
  - to verify the transfer of assets (safeguarded again counterfeiting of currency).
- It by-passes centralised regulatory controls and uses blockchain, a public transaction database.
- It is not formally backed up by legal entity as fiat currency.
- Its perceived anonymity has once made cryptocurrency popular in illegal goods trading.

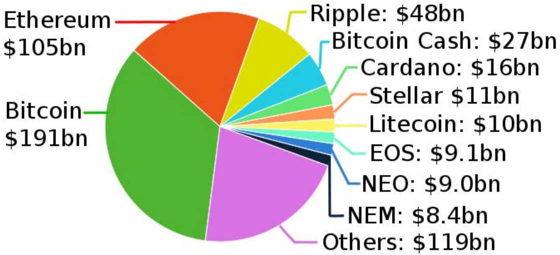


When is cryptocurrency created?

- Cryptographic protocols start from early 1980s on anonymous communication (Chaum 1981).
- Its creation is perhaps a libertarian response to the central bank failure to manage the credit bubbles with barely a fraction in reserve in 2008.
- In October 2008, Nakamoto published a paper titled "Bitcoin: A Peer-to-Peer Electronic Cash System".
- Bitcoin is the first cryptocurrency created in 2009.
- It also competes against online payment methods, e.g. PayPal.

How is cryptocurrency now?

- There are more than 2800 cryptocurrencies currently, some have just created and some have already exited.



- Total market capitalization:
  more than 600 billion USD in Dec 2017.

How is cryptocurrency now?

- It is increasingly accepted by banks such as UBS and Credit Suisse.
- Bitcoin futures are traded now.
- Technological factors: Security & confirmation time.
  - Ethereum and Dash: have near instant transaction using technologies smart contract or InstantSend.
  - Bitcoin and NEM: relatively slower and hence has higher liquidity risk.
- Crytpocurrency is in the early stage of development but it has the potential to challenge fiat currency.

Its role as a currency or speculative asset?

- Recently, cryptocurrency market is extremely volatile with also fast changing cryptocurrency communities.

  E.g. Bitcoin

- Hence its role as a currency, that is, as medium of exchange, units of account and stores of value, is questioned.
- Recently Australian government removed GST from cryptocurrency, so it is treated as a currency.

State of the Art

- Many studies were directed to the technological, economical and legal aspects of cryptocurrencies.
- Statistical models are still basic and immature. Examples,
  - **Speculation and volatility**: Regression and GARCH models

    To study the determinants of cryptocurrency as a currency on market stability, competition, efficiency, price driver, technical factors, turnover and even Google search counts.
  - **Persistence and predictability:** Hurst exponential and fractional integrated model for both returns & volatility, etc.
- They mainly focus on Bitcoin, hence fails to detect the cross dependency for a basket of cryptocurrencies in portfolio setting.

Data of 224 cryptocurrencies

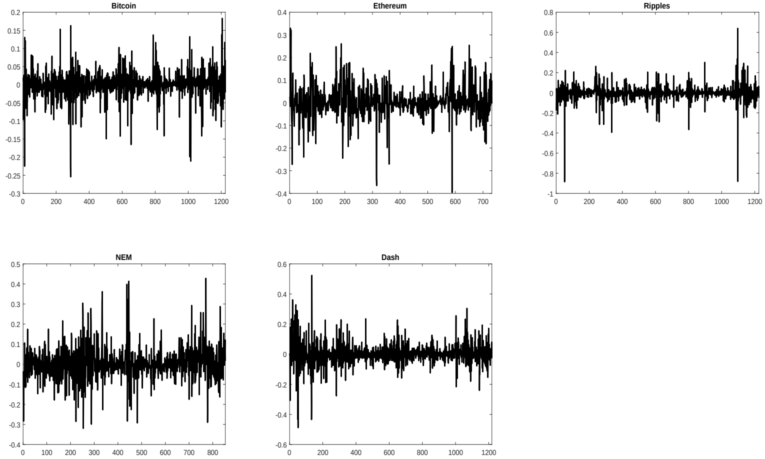
- From the Brave New Coin (BNC) Digital Currency indices.
- We consider return as the daily price percentage change $y_t = (P_t - P_{t-1})/P_{t-1}$ for 224 cryptocurrencies, exchanged at least once per day since inception.
- We further focus on top 5 cryptocurrencies by market capitalization on July 31, 2017.

Summary statistics for the top 5 cryptocurrencies

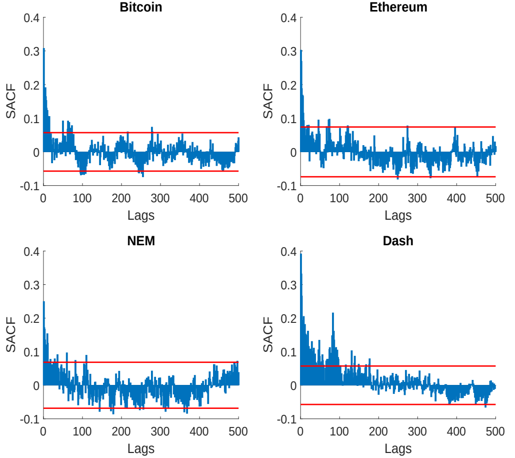
	Cap	No	Mean	Std.	Skew	Kur	Min.	Max.	LB1	LB2	NT
BTC	67.76	1225	0.0009	0.0362	-1.046	11.96	-0.254	0.183	439	212	4320
ETH	28.50	732	0.0054	0.0742	-0.098	7.43	-0.396	0.329	244	122	600
RIP	6.77	1225	-0.0003	0.0751	-2.085	40.70	-0.884	0.639	460	89	73427
NEM	2.36	853	0.0046	0.0831	0.526	7.07	-0.320	0.428	227	136	629
Dash	1.50	1219	0.0020	0.0724	0.083	11.75	-0.488	0.523	903	587	3887

LB: Ljung-Box test for residual autocorrelation. LB1: for $|y_t|$; LB2: test y_t^2
P-values for all LB test and normality test (NT) are $< .0001$.

History plot of y_t



ACF plot of $|y_t|$ for volatility



Oscillating behaviour!

Bitcoin displays the most oscillating behaviour in ACF.

SV models with Gegenbauer long memory (GLM), leverage (LVG) and heavy tails (HT)

- We model $y_t, t = 1, 2, ..., T$ by a stochastic process:

$$\text{GLM} : \quad y_t = (1 - 2uB + B^2)^{-d} \varepsilon_t = \sum_{j=0}^{\infty} \lambda_j \varepsilon_{t-j},$$

$$\text{SV} : \quad h_{t+1} = \alpha + \beta(h_t - \alpha) + \eta_{t+1},$$

$$\text{LVG-HT} : \quad \begin{pmatrix} \varepsilon_t \\ \eta_{t+1} \end{pmatrix} \sim t_\nu \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} e^{h_t} & \sigma \rho e^{h_t/2} \\ \sigma \rho e^{h_t/2} & \sigma^2 \end{pmatrix} \right).$$

where h_t is the log-volatility, α is the constant level of volatility, β ($|\beta| < 1$) is the persistence of volatility and σ^2 is the volatility of volatility.

- y_t has long memory effects when

$$(\{|u| < 1, 0 < d < 0.5\} \cup \{|u| = 1, 0 < d < 0.25\}).$$

SV models with Gegenbauer long memory (GLM), leverage (LVG) and heavy tails (HT)

- It is a MA(∞) process approximated by a truncated MA(J).
- The Gegenbauer coefficients λ_t satisfy $\lambda_0 = 1, \lambda_1 = 2ud$ and the recursion

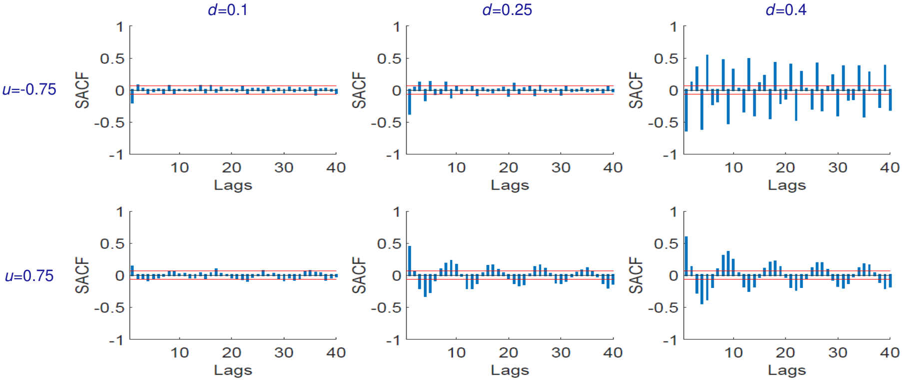
$$\lambda_j = 2u\left(\frac{d-1}{j}+1\right)\lambda_{j-1} - \left(\frac{2(d-1)}{j}+1\right)\lambda_{j-2}, \quad j \geq 2.$$

- The leverage effect between errors of y_t & h_{t+1} is $\rho = \mathbb{E}[\varepsilon_t \eta_{t+1}]$.
- The bivariate t error distribution is expressed in SMN:

$$\begin{pmatrix} \varepsilon_t \\ \eta_{t+1} \end{pmatrix} \sim \text{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \xi_{t+1}\begin{pmatrix} e^{h_t} & \sigma\rho e^{h_t/2} \\ \sigma\rho e^{h_t/2} & \sigma^2 \end{pmatrix}\right)$$

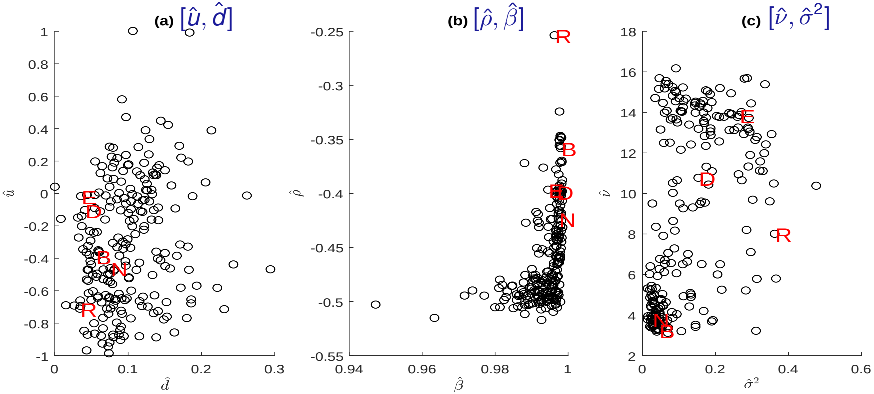
where $\xi_{t+1} \sim \text{IG}\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$.

ACF across levels of *u* and *d*



- Larger *d* is more persistent and invokes clearer cyclic ACF.
- Positive *u* has smoother autocorrelation cycle.
  Negative *u* has instantaneously oscillating autocorrelation patterns.

Parameter estimates for 224 cryptocurrencies using Bayesian MCMC



Scatter plots of parameter estimates for 224 cryptocurrencies under the GLM-SV-LVG-HC model.

B: Bitcoin, E: Ethereum, R: Ripples, N: NEM, D: Dash.

Properties of 224 cryptocurrencies

- Lower persistence d implies lower predictability:
  Moderate in general.
  Top 5 all have weak long memory.
- ACF instantaneously oscillating:
  Most u are negative.
  Top 5 are all negative but Ethereum & Dash are close to 0.
- High volatility persistence: All β close to 1.
- Leverage effect: Moderate and ρ cluster around -0.5.
- Two volatility groups depending on ν and σ^2.

Two distinct groups in volatility features

- Two groups:

  Gp 1: Low σ^2 but high kurtosis ($\nu < 7$).

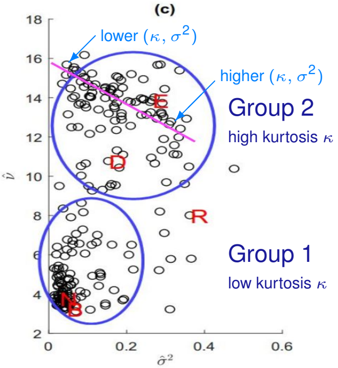
  Gp 2: High σ^2 but low kurtosis ($\nu > 12$).

- The wild nature of cryptocurrency is due to

  higher kurtosis (Gp 1) or
  higher σ^2 (Gp 2)
  in their error distributions,
  but not both.

- Within group 2,

  higher σ^2 goes with higher kurtosis
  lower σ^2 goes with lower kurtosis



Technological factor for top 5

- Bitcoin & NEM tends to be a group & Dash & ETH another.
- Ripple is quite distinct: lowest leverage ρ & long memory d.
  - Users can store any fiat/cryptocurrency asset on the network, so is the only currency insulated from future exchange volatility & has no counter-party credit risk.
  - Due to this safety feature, Ripples is increasingly used by banks and large corporations as the preferred settlement infrastructure.
- Dash & ETH have faster transaction & lower liquidity risk:

  In lower kurtosis group with lighter tails.

  Weaker long memory; so less predictability.
- Bitcoin & NEM have slower transaction:

  In higher kurtosis group with heavier tails.

  Relatively stronger oscillating long memory & higher predictability.

Model comparison for Bitcoin

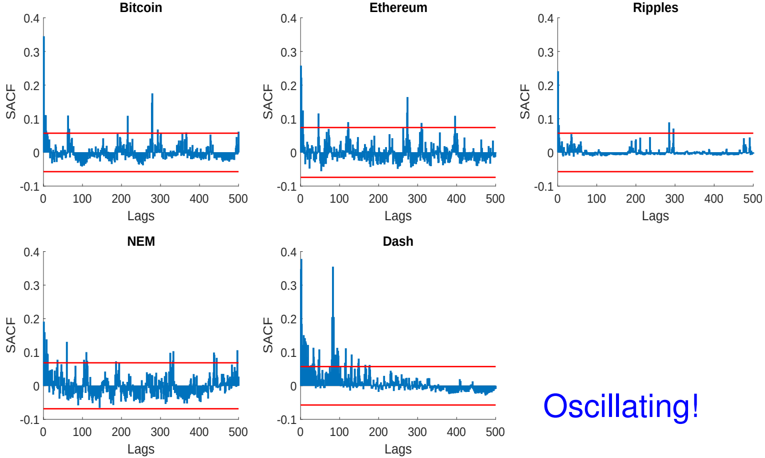
Parameter estimates of six in-sample fitted models for Bitcoin.

Model	u	d	α	β	σ^2	ν	ρ	DIC
SV			-7.690	0.864	0.520			-12285.5
SV-LVG			-7.048	0.964	0.103		-0.301	-12808.8
SV-GMA	-0.389	0.048	-7.724	0.853	0.579			-12366.2
SV-GMA-LVG	-0.396*	0.021*	-7.039	0.964	0.102		-0.298	-12827.9
SV-GMA-HC	-0.394	0.056	0.0002*	0.998	0.046	3.327		-15745.4
SV-GMA-LVG-HC	-0.379	0.053	-0.003*	0.998	0.049	3.263	-0.372	-15973.5

GMA: Gengenbauer long memory; LVG: leverage; HC: heavy tails.
Estimate with $*$ is insignificant.

- With leverage effect, volatility of volatility $\hat{\sigma}^2$ drops.
- With heavy tails to allow for the distorting effects of outliers,
  - Volatility of volatility $\hat{\sigma}^2$ drops further.
  - Volatility persistence $\hat{\beta}$ increases.
  - Volatility level $\hat{\alpha}$ increases to 0.
  - Return persistence gets stronger.
  - DIC improves significantly.

ACF plot of y_t^2 for volatility



Oscillating!

Ripples has the least while Dash has the most persistence. Others are clearly oscillating.

Volatility measure

- However, volatility measure such as (realised) daily range is more efficient than the latent volatility h_t in the SV model.
- We define a volatility measure on real support as

$$v_t = \log(R_{h,t} - R_{l,t}),$$

- The high and low daily returns are $R_{k,t} = (P_{k,t} - P_{c,t-1})/P_{c,t-1}, k = h, l$ respectively.
- Consistent with the return as daily price percentage change $y_t = (P_t - P_{t-1})/P_{t-1}$ and $P_{k,t}, k = h, l, c$ represents the high, low and closing price of day t.

SV model with buffer threshold and jumps for returns & Gegenbauer long memory for volatility measures

- Moreover, long memory may be confused with regime changes in returns (Guegan, 2005).
- The ACF of y_t^2 confirms the presence of oscillating long memory for volatility.
- Assign AR buffer threshold with jumps (ARBJ) to return y_t & Gegenbauer long memory (GLM) to volatility measure v_t.

$$\text{ARBJ:}\ \ y_t = \begin{cases} \phi_1 y_{t-1} + k_t q_t + e_t, & \text{if } R_t = 1, \\ \phi_2 y_{t-1} + k_t q_t + e_t, & \text{if } R_t = 0, \end{cases} \qquad e_t \sim N(0, e^{h_t})$$

$$\text{SV:}\ \ h_t = \alpha + \beta(h_{t-1} - \alpha) + \eta_t, \qquad \eta_t \sim N(0, \sigma^2),$$

$$\text{GLR:}\ \ v_t = (1 - 2uB + B^2)^{-d}(\gamma + h_t + \xi_t), \qquad \xi_t \sim N(0, \sigma_v^2),$$

$$\text{Initial: } h_1 \sim \mathcal{N}\left(\alpha, \frac{\sigma^2}{1-\beta^2}\right) \ \&\ y_1 \sim \begin{cases} \mathcal{N}\left(\frac{k_1 q_1}{1-\phi_1}, \frac{e^{h_1}}{1-\phi_1^2}\right), & \text{if } R_1 = 1, \\ \mathcal{N}\left(\frac{k_1 q_1}{1-\phi_2}, \frac{e^{h_1}}{1-\phi_2^2}\right), & \text{if } R_1 = 0, \end{cases}$$

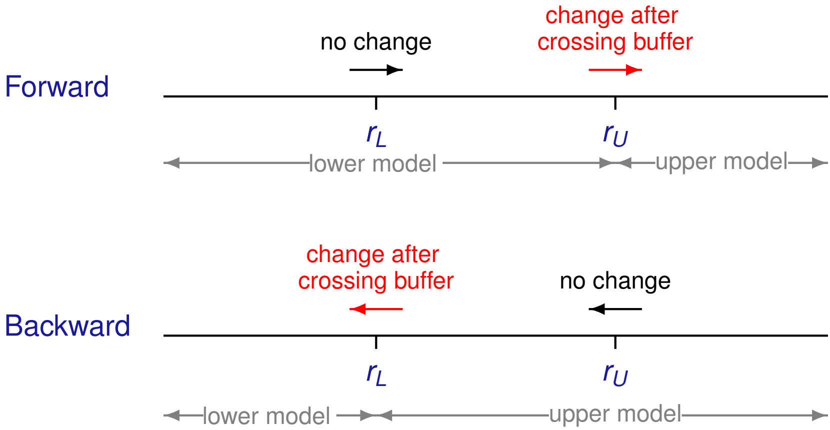
SV model with buffer threshold with jumps for returns & Gegenbauer long memory for volatility measures

- Additionally, γ is the level of the volatility measure, and σ_v^2 is the volatility of the volatility measure.
- The buffer regime indicators

$$R_t = \begin{cases} 1, & \text{if } y_{t-\tau} \le r_L, \\ R_{t-1}, & \text{if } r_L < y_{t-\tau} \le r_U, \ \text{(buffer region)} \\ 0, & \text{if } y_{t-\tau} > r_L. \end{cases}$$

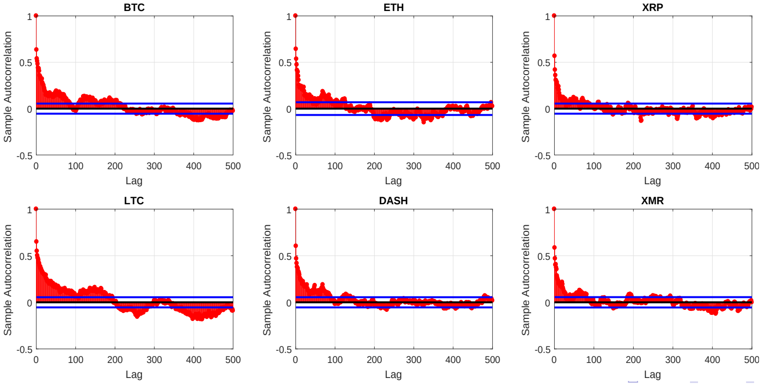
- The jump indicator $q_t \in \{0, 1\}$ has probability $\mathbb{P}(q_t = 1) = \pi_q$ and the jump size $k_t \sim \mathcal{N}(\mu_k, \sigma_k^2)$.
- For leverage effect, $\gamma_1 y_{t-1} + \gamma_2 |y_{t-1}|$ can be added to h_t and/or v_t models conditionally.

Buffer threshold



Data of 149 cryptocurrencies

- A slightly different data with 149 cryptocurrencies ended on Dec 31, 2017.
- The ACFs of volatility measure v_t for the top 6 by market capitalisation on Dec 31, 2017 again show oscillations.



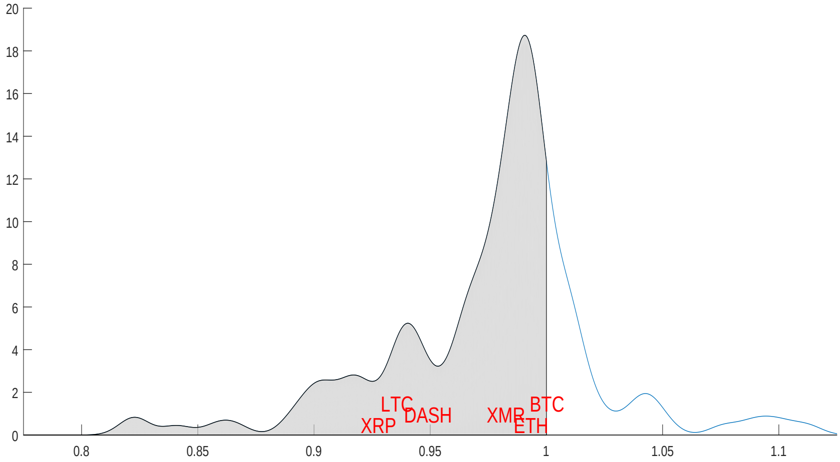
New results

	r^U	r^L	ϕ^U	ϕ^L	u	d	α	β	σ^2	γ	σ_v^2	π_q	μ_k	σ_k^2
B	0.054	-0.20	0.022	-0.65	=1	0.001	-7.52	0.65	0.41	4.03	0.005	0.001	0.024	2.04
	0.003	-0.19	0.021	-0.21	-0.77	0.211	-7.55	0.79	0.40	2.95	0.005	0.001	0.003	2.23
E	0.012	-0.13	0.038	-0.38	=1	0.000	-6.00	0.68	0.37	3.30	0.007	0.002	0.023	2.10
	-0.001	-0.12	0.034	-0.36	-0.79	0.240	-6.08	0.82	0.37	2.38	0.006	0.002	0.020	2.10
X	0.019	-0.13	-0.030	-0.18	=1	0.001	-6.44	0.60	0.47	3.62	0.007	0.004	0.876	0.97
	-0.041	-0.10	0.006	-0.18	-0.76	0.227	-6.51	0.76	0.48	2.68	0.005	0.002	1.144	1.29
L	0.072	-0.22	-0.021	-0.09	=1	0.001	-7.11	0.67	0.49	3.84	0.006	0.012	0.298	0.01
	0.001	-0.08	-0.010	-0.09	-0.85	0.282	-7.21	0.84	0.50	2.32	0.005	0.010	0.259	0.02
D	0.016	-0.20	0.014	-0.36	=1	0.001	-6.12	0.63	0.32	3.48	0.006	0.004	0.596	0.43
	-0.005	-0.20	0.003	-0.26	-0.75	0.328	-6.17	0.82	0.34	2.00	0.004	0.001	0.084	2.03
M	-0.023	-0.19	-0.019	-0.45	=1	0.001	-5.74	0.61	0.25	3.36	0.005	0.002	0.424	0.92
	0.003	-0.19	-0.017	-0.33	-0.86	0.173	-5.76	0.75	0.26	2.73	0.005	0.001	0.097	1.99

B: BTC, Bitcoin; E: ETH, Ethereum; **X: XRP, Ripple**; L: LTC, Litecoin; D: Dash; M: XMR, Monero

- GLM for volatility measure consistently gives better fit than standard LM and stronger LM.
- It consistently gives lower γ, the level of volatility measure.

Ratio of DIC for GLM to standard LM models



Ratio of DIC for GLM to standard LM models

- Majority, 118 (79%) of cryptocurrencies favour GLM with oscillating ACF (ratio < 1).
- They have less dampening ACF behaviour.
- Bitcoin, Ethereum and Monero (in order) have higher ratio & weaker preference for GLM.
- Dash, Litecoin and Ripple (in order) have lower ration & stronger preference for GLM.
- Only Litecoin has both significant jump probability and jump size.
- Ripple again shows the lowest ratio so the most favour for GLM.

Multivariate skew variance gamma (MSVG) distribution

- Previous models consider one cryptocurrency at a time.
- To model a basket jointly, we consider MSVG distribution with pdf:

$$f_{VG}(\boldsymbol{y}) = 2^{1-\nu}\pi^{-\frac{d}{2}}|\boldsymbol{\Sigma}|^{-\frac{1}{2}}\frac{\nu^{\frac{d}{2}}}{\Gamma(\nu)}\frac{K_{\nu-\frac{d}{2}}\left(\sqrt{[2\nu+\boldsymbol{\gamma}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\gamma}](\boldsymbol{y}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{y}-\boldsymbol{\mu})}\right)\exp\left((\boldsymbol{y}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}\boldsymbol{\gamma}\right)}{[(2\nu+\boldsymbol{\gamma}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\gamma})(\boldsymbol{y}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{y}-\boldsymbol{\mu})]^{-\frac{2\nu-d}{4}}[1+\frac{1}{2\nu}\boldsymbol{\gamma}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\gamma}]^{\frac{2\nu-d}{2}}}$$

where

$\boldsymbol{\mu} \in \mathbb{R}^d$ is the location parameter,

$\boldsymbol{\Sigma}$ is a $d \times d$ positive definite symmetric scale matrix,

$\boldsymbol{\gamma} \in \mathbb{R}^d$ is the skewness parameter,

$\nu > 0$ is the shape parameter,

$\Gamma(\cdot)$ is the gamma function and

$K_\eta(\cdot)$ is the 2nd kind modified Bessel function with index η.

MSVG distribution

- It has a mean-variance normal mixtures representation:

$$\boldsymbol{y}_i|\lambda_i \sim \mathcal{N}_d(\boldsymbol{\mu} + \boldsymbol{\gamma}\lambda_i, \lambda_i\boldsymbol{\Sigma}), \quad \lambda_i \sim \mathcal{G}(\nu, \nu)$$

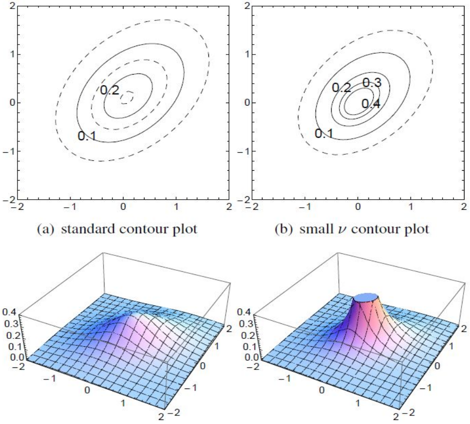
  where $\mathcal{G}(\alpha, \beta)$ is a Gamma distribution and $\boldsymbol{y}_i$ are returns.

- The mean and variance are:

$$\mathbb{E}(\boldsymbol{Y}_i) = \boldsymbol{\mu} + \boldsymbol{\gamma} \quad \text{and} \quad \mathbb{C}\text{ov}(\boldsymbol{Y}_i) = \boldsymbol{\Sigma} + \frac{1}{\nu}\boldsymbol{\gamma}\boldsymbol{\gamma}'.$$

- When $\nu \leq \frac{d}{2}$, the kurtosis is high and the pdf at $\boldsymbol{\mu}$ becomes unbounded.

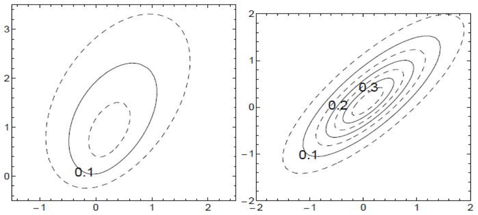
Densities of MSVG distribution



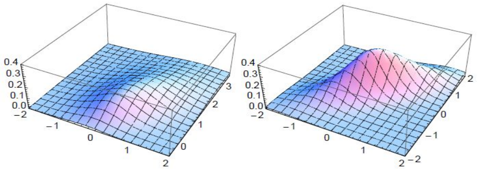
(a) standard contour plot (b) small ν contour plot

(c) standard 3D plot (d) small ν 3D plot

Densities of MSVG distribution



(e) large γ contour plot (f) large correlation contour plot



(g) large γ 3D plot (h) large correlation 3D plot

Data of four cryptocurrencies

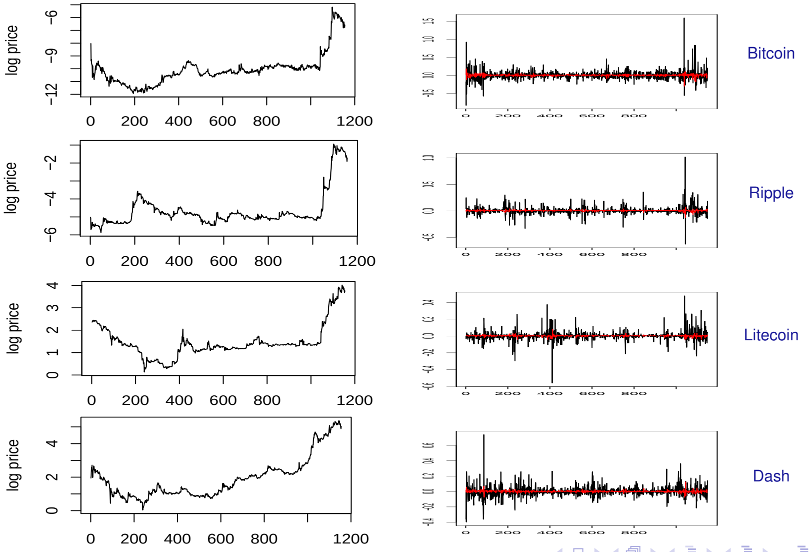
- We take Bitcoin, Ripple, Litecoin and Dash during May 21, 2014 to July 17, 2017 for the days when all 4 are observed.
- Correlation:
  Litecoin & Dash are most and Bitcoin & Dash are least.

	Bitcoin	Ripple	Litecoin	Dash
Bitcoin	1.000	0.088	0.131	0.010
Ripple		1.000	0.146	0.029
Litecoin			1.000	0.279
Dash				1.000

- Summaries and *P*-value of Box-Pierce test for serial correlation.

	median	mean	SD	skewness	kurtosis	Box-Pierce
Bitcoin	−0.0018	0.0013	0.133	1.25	37.6	3.6e-9
Ripple	−0.0022	0.0029	0.076	1.68	41.7	0.0015
Litecoin	−0.0003	0.0012	0.055	0.40	23.7	0.1992
Dash	−0.0021	0.0027	0.070	1.19	17.9	0.6779

Plots of log price and return



Vector ARMA-MSVG model

- Vector ARMA-MSVG(p, q) model

$$\boldsymbol{y}_t = \boldsymbol{c} + \boldsymbol{A}_1 \boldsymbol{y}_{t-1} + \cdots + \boldsymbol{A}_p \boldsymbol{y}_{t-p} - \boldsymbol{B}_1 \hat{\varepsilon}_{t-1} - \cdots - \boldsymbol{B}_q \hat{\varepsilon}_{t-q} + \varepsilon_t$$

  where
  $\varepsilon_t \sim \mathcal{VG}_d\left(-\gamma, \boldsymbol{\Sigma}, \gamma, \nu\right)$, $\boldsymbol{c} \in \mathbb{R}^d$,
  $\boldsymbol{A}_1, \ldots, \boldsymbol{A}_p \in \mathbb{R}^d \times \mathbb{R}^d$ are the AR coefficient matrices and
  $\boldsymbol{B}_1, \ldots, \boldsymbol{B}_q \in \mathbb{R}^d \times \mathbb{R}^d$ are the MA coefficient matrices.

- The model can also be expressed as

$$\boldsymbol{y}_t' = \boldsymbol{x}_t' \beta + (\varepsilon_t + \gamma)'$$

  where
  $\beta' = \begin{pmatrix} \boldsymbol{c} & \boldsymbol{A}_1 & \cdots & \boldsymbol{A}_p & \boldsymbol{B}_1 & \cdots & \boldsymbol{B}_q \end{pmatrix}$ is a $d \times [d(p+q)+1]$ matrix,
  $\boldsymbol{x}_t' = \begin{pmatrix} 1 & \boldsymbol{y}_{t-1}' & \cdots & \boldsymbol{y}_{t-p}' & -\varepsilon_{t-1}' & \cdots & -\varepsilon_{t-q}' \end{pmatrix}$ is a $[d(p+q)+1]$ vector,

Parameter estimation for VARMA-MSVG model

- The error terms ε_t depends on $\boldsymbol{c}, \boldsymbol{A}_1, ..., \boldsymbol{A}_p, \boldsymbol{B}_1, ..., \boldsymbol{B}_q$ so the parameter estimation is complicated.
- Two-stage linear approximation method:

  Stage 1: Approximate the error terms by fitting the series with a high order Vector AR model using Expectation Conditional Maximisation (ECM).

  Stage 2: Use these fitted errors to estimate other model parameters.

AICc	$q = 0$	$q = 1$	$q = 2$	$q = 3$
$p = 0$	-13287	-13325	-13374	-13357
$p = 1$	-13314	-13328	-13188	
$p = 2$	$\mathbf{-13384}$	-13187		
$p = 3$	-13356			

Results for VARMA-MSVG model

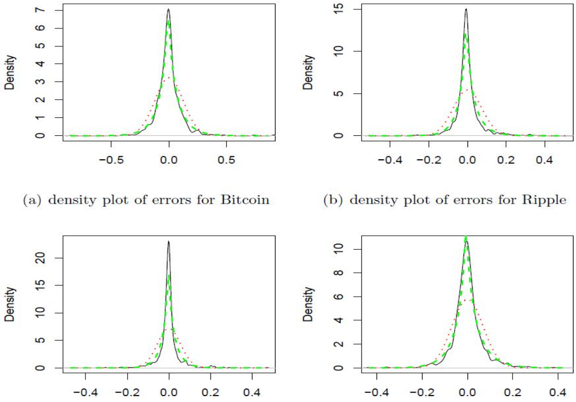
Estimate and SE of VARMA-VG(2,0) model

Par	Estimate	SE
μ^T	$\begin{pmatrix} -0.004 & -0.003 & -0.001 & -0.003 \end{pmatrix}$	$\begin{pmatrix} 0.002 & 0.001 & 0.001 & 0.001 \end{pmatrix}$
A_1	$\begin{pmatrix} -0.191 & 0.063 & 0.054 & 0.079 \\ -0.003 & -0.131 & 0.013 & 0.004 \\ -0.007 & 0.003 & -0.072 & 0.007 \\ -0.010 & -0.039 & -0.048 & -0.013 \end{pmatrix}$	$\begin{pmatrix} 0.031 & 0.043 & 0.059 & 0.083 \\ 0.010 & 0.026 & 0.036 & 0.036 \\ 0.011 & 0.014 & 0.026 & 0.011 \\ 0.014 & 0.024 & 0.026 & 0.018 \end{pmatrix}$
A_2	$\begin{pmatrix} -0.069 & 0.075 & -0.037 & 0.016 \\ -0.026 & -0.028 & 0.003 & -0.038 \\ 0.003 & 0.043 & -0.129 & 0.022 \\ -0.049 & 0.077 & -0.030 & -0.084 \end{pmatrix}$	$\begin{pmatrix} 0.006 & 0.034 & 0.048 & 0.037 \\ 0.002 & 0.022 & 0.025 & 0.014 \\ 0.002 & 0.012 & 0.021 & 0.010 \\ 0.002 & 0.021 & 0.025 & 0.010 \end{pmatrix}$
Σ	$\begin{pmatrix} 0.0138 & 0.0010 & 0.0008 & 0.0004 \\ & 0.0038 & 0.0005 & 0.0006 \\ & & 0.0021 & 0.0008 \\ & & & 0.0051 \end{pmatrix}$	$\begin{pmatrix} 0.0009 & 0.0002 & 0.0002 & 0.0003 \\ & 0.0002 & 0.0001 & 0.0001 \\ & & 0.0001 & 0.0001 \\ & & & 0.0003 \end{pmatrix}$
γ^T	$\begin{pmatrix} 0.007 & 0.006 & 0.002 & 0.006 \end{pmatrix}$	$\begin{pmatrix} 0.003 & 0.002 & 0.001 & 0.002 \end{pmatrix}$
ν	0.7447	0.0333

Results for VARMA-VG model

- **Volatility:** Litecoin has lowest σ^2 & Bitcoin has highest σ^2, all agree with observation.
- **Skewness:** Litecoin has lowest γ, agree with observation.
- **Persistence:** Litecoin & Dash are weaker as they have lower diagonals of $\boldsymbol{A}_1$.
- **Cross-dependence:** Litecoin & Dash are more correlated, again agree with observation and previous result.
- **Leptokurtosis:** as $\nu < d/2 = 2$, the density is unbounded.

Model-fit: black observed; green VG; red normal



(a) density plot of errors for Bitcoin
(b) density plot of errors for Ripple
(c) density plot of errors for Litecoin
(d) density plot of errors for Dash

black line: observed
green dash line: fitted by VG
red dotted line: fitted by normal

Consistent kurtosis with same df: Too restrictive.
Ripple and Litecoin have higher peak. Normal is worst.

Conclusion

- Interestingly, technological (not economic) factor distinguishes the behaviour of cryptocurrency.
- Faster transaction gives lower liquidity risk, hence lower leverage, stronger GLM and lighter tails.
- MSVG distribution can also show cross dependency apart from technological factor.
- Two groups:
  - **Faster transaction group:** Litecoin, Dash & Ethereum.
  - **Slower transaction group:** Bitcoin, NEM & Monero.
- Ripple is very distinct as the only currency with no over night risk.

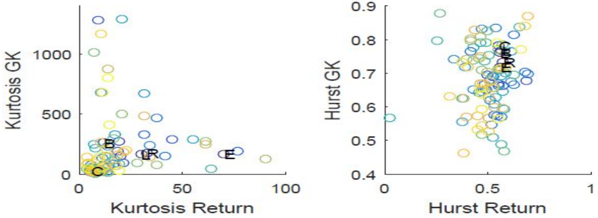
Future research

- **Extension I:** Include leverage effect and heavy tail distribution, eg t & VG. Complicated as with mixing variables, there will be many model parameters.
- **Extension II:** The VARMA-MSVG provides reasonable fit to the 4 cryptocurrencies jointly. Some possible extensions:
  - Include ARIMA, ARFIMA or GLM and allow different df. Challenging!
  - Add volatility measures in a 2-stage model:
    Stage 1: Model volatilities using CARR model and then
    Stage 2: Insert fitted volatilities as $\hat{\sigma}^2_{t,ij}$ to estimate the multivariate model.
  - Improve the efficiency of volatility measures with various types.
- There are many promising model choices to explore.

Garman Klass volatility measure and return

Garman Klass (1980) proposed unbiased volatility measure:

$$\sigma^2_{GK} = 0.5(H_t - L_t)^2 - (2\ln 2 - 1)(C_t - O_t)$$



The CARR model for volatility measure and return model are

CARR: $V_{GK} = \lambda_t \eta_t, \quad \eta_t \sim GB2(a, b_t(\lambda_t), p, q)$

CARR: $\lambda_t = \beta_0 + \beta_{11} v_{t-1} + \beta_{21} \lambda_{t-1} + \beta_{31} v_{t-1} \lambda_{t-1} + \beta_4 |y_{t-1}| + \beta_5 y_{t-1}$

Return: $y_t = \mu_t + \hat{\lambda}_t \varepsilon_t, \quad \varepsilon_t \sim VG(0, \nu)$

Reference

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- Nitithumbundit, T. (2017). EM Algorithms for Multivariate Skewed Variance Gamma Distribution with Unbounded Densities and Applications. Thesis, The University of Sydney, 123-129.