A theoretical framework for extracting some temporal and frequency features in non-stationary fractional signals.

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ISM: Institute of Statistical Mathematics Tokyo Japan

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- **Illustration:** present a worked example and other possible future applications...

Notation and Models...

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The wavelet method is the most suitable in this setting

The notion of localised energy...

Let ψ be a filter having a Fourier transform concentrated on $\mathbb{B} = [\omega_1, \omega_2]$ and let us define :

$$W(s) \ = \int_{\mathbb{R}} \psi(t-s)\ X(t)\ dt$$

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The alternative is to introduce the concept of pseudo compactness of the support ....

The concept of the ρ-pseudo compact support

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We will say that g have a ρ-pseudo support $\mathbb{I}$ if

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A simple general framework example

Let ψ be a fixed filter, a kind of a *mother wavelet* with,

- A temporal support $[L_1,\ L_2]$ i.e. $\psi(t) = 0$ for $t \notin [L_1,\ L_2]$
- And a frequency ρ-pseudo support $[\Lambda_1,\ \Lambda_2]$

$\Longrightarrow$ For a given frequency band, $\mathbb{B} = [\omega_1,\ \omega_2]$, we can build a wavelet having a targeted ρ-pseudo support $\mathbb{B}$.

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The underlying modulation parameter η and the scaling parameter λ will depends on :

- The targeted band $\mathbb{B}$ bounds ω_1 and ω_2.
- The mother wavelets ψ parameters Λ_1 and Λ_2.

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Using the fact that:

$$\hat{\psi}_1(\xi) = \hat{\psi}(\frac{\xi - \eta}{\lambda})$$

We deduce then that :

$$\rho - \textit{pseudo supp of } \hat{\psi}_1 = \eta + \lambda \times \rho - \textit{pseudo supp of } \hat{\psi}$$

A simple algebra imply that:

$$\lambda = \frac{\omega_2 - \omega_1}{\Lambda_2 - \Lambda_1}$$
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In addition the temporal support of ψ_1 is given by:

$$\left[\frac{\Lambda_2 - \Lambda_1}{\omega_2 - \omega_1} L_1 \quad , \quad \frac{\Lambda_2 - \Lambda_1}{\omega_2 - \omega_1} L_2\right]$$

Symmeric examples: Gabor wavelets

The Gabor mother wavelet through the Gauss- Laplace function:

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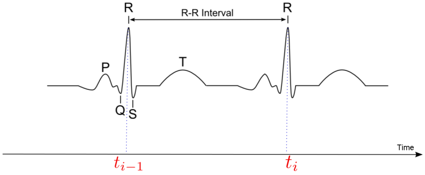
We obtain the parameters η and σ as

$$\eta = \frac{\omega_1 + \omega_2}{2} \qquad \text{and} \qquad \sigma = \frac{2L}{\omega_2 - \omega_1}$$

In addition $|\rho \ pseudo \ supp \ \psi| = \dfrac{4L^2}{\omega_2 - \omega_1}$

A worked example: physiological data

Let us denote $(t_i)_{i=1,\ldots,N}$, instants corresponding to R pics. We consider the RR-time series: $X(t_i) = (t_i - t_{i-1})$.



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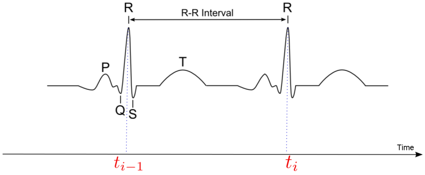

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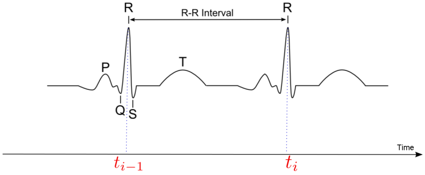

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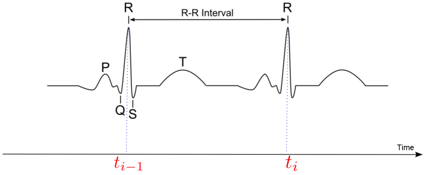

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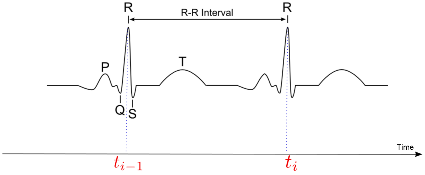

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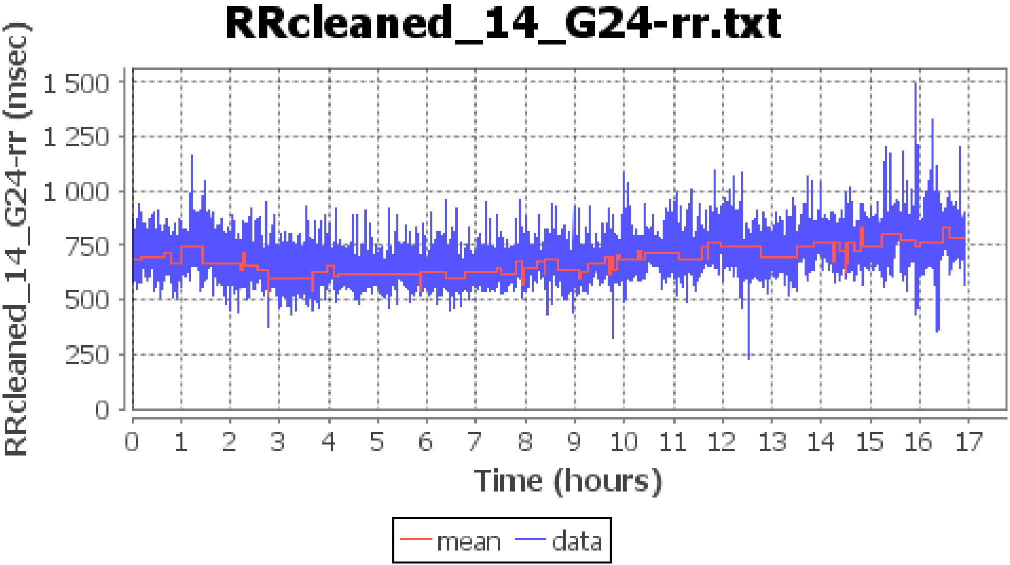
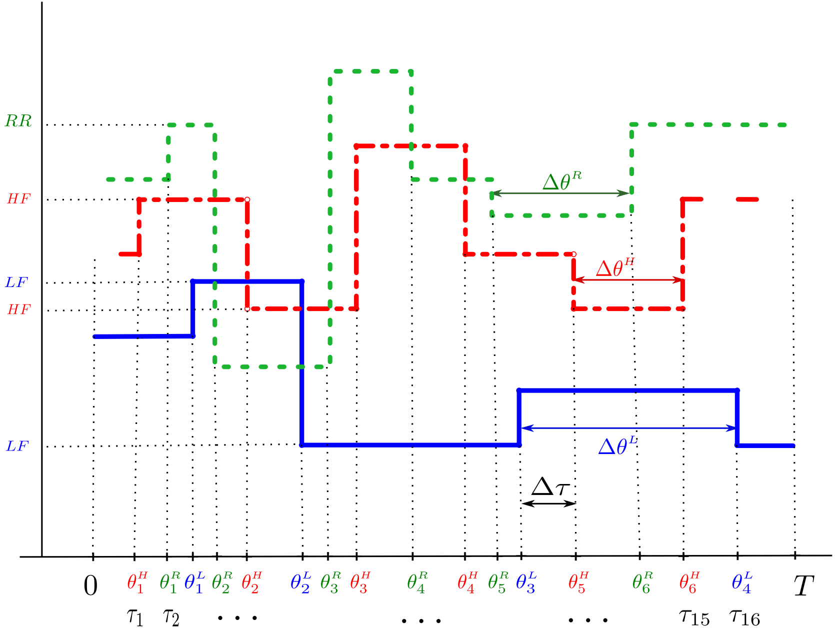
Hence the importance of extracting HF and LF energies in this problem.

A sample observed signal



Discriminant features construction



We will be interested in the forthcoming variables:

- The variable $\theta_i^R - \theta_{i-1}^R$ which represents the time lapse where the RR signal has a stationary behaviour.
- The variable $\theta_i^H - \theta_{i-1}^H$ which represents the duration of the i^{th} level of the HF energy.

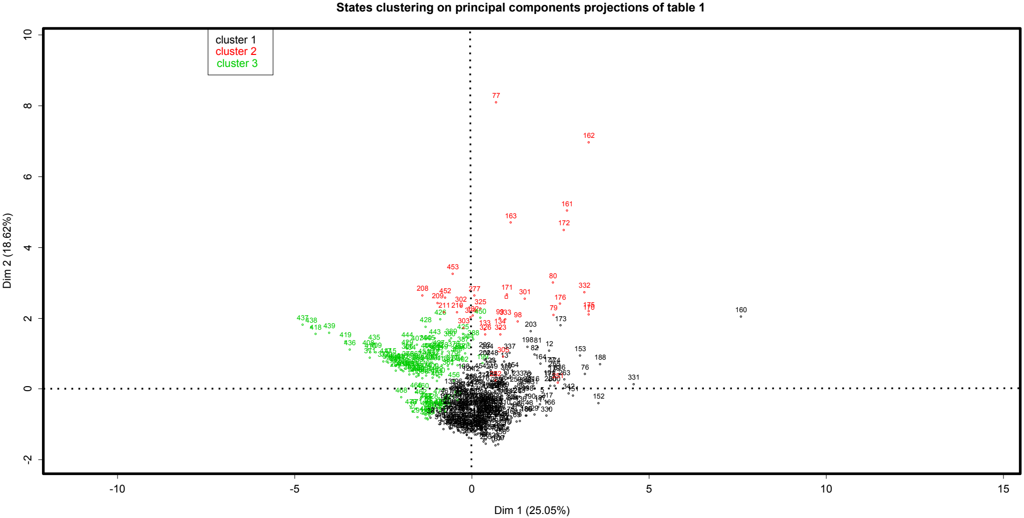
  $\Longrightarrow$ This duration can be seen as the duration where only the parasympathetic (braking) system is activated and has a fixed regime.
- The variable $\theta_i^L - \theta_{i-1}^L$ which represents the duration of the i^{th} level of the LF energy.

  $\Longrightarrow$ This duration can be seen as the lapse of time where only the orthosympathetic (acceleration) system is in action and has established a fixed regime.
- the variable $\tau_i - \tau_{i-1}$ which represents the inter RR, HF and LF durations of the i^{th} level of the HF energy before one of the two systems switches to another state.

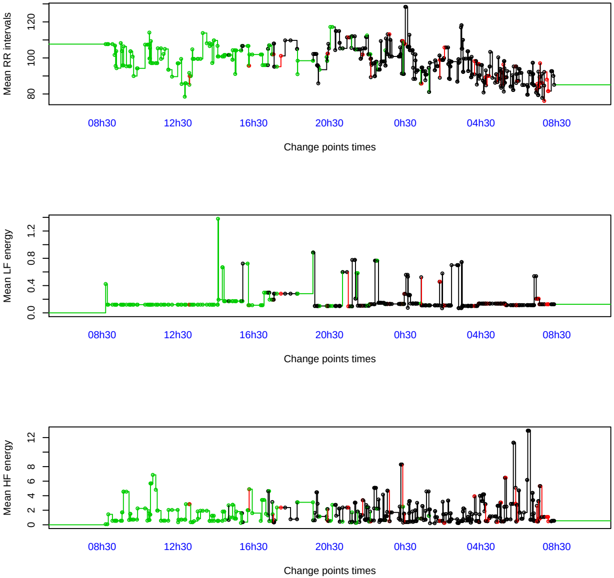
Discriminant features table

States	$\Delta\theta^R$	$\Delta\theta^L$	$\Delta\theta^H$	$\Delta\tau$	RR	LF	HF
1	$\theta_2^R - \theta_1^R$	$\theta_2^L - \theta_1^L$	$\theta_2^H - \theta_1^H$	$\tau_2 - \tau_1$	RR_1	LF_1	HF_1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
M-1	$\theta_M^R - \theta_{M-1}^R$	$\theta_M^L - \theta_{M-1}^L$	$\theta_M^H - \theta_{M-1}^H$	$\tau_M - \tau_{M-1}$	RR_{M-1}	LF_{M-1}	HF_{M-1}
M	$T - \theta_M^R$	$T - \theta_M^L$	$T - \theta_{M-1}^H$	$T - \tau_M$	RR_M	LF_M	HF_M

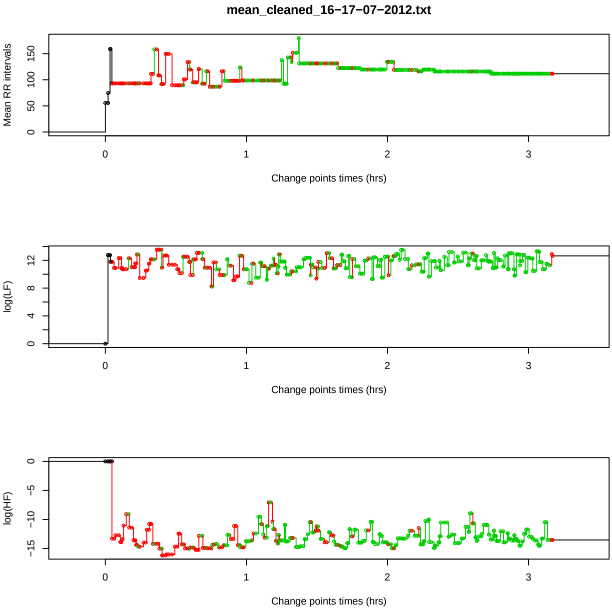
Clustering projection of features



An example of clustered signal



An other example on animals behaviour



Other possible applications

- Application other physiological signals, and behavioural analysis...
- Pattern detection in some complex temporal signals.
- Event and Fault detection in some complex power mechanical systems.
- Detection of some hidden structures, noises, in speech signals.